Fertility and Childlessness in the United States

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Abstract

We develop a theory of fertility, distinguishing the choice to have children from the choice of the number of children. The deep parameters of the model are identified using three facts of the 1990 US Census: (1) fertility of mothers decreases with education; (2) childlessness exhibits a U-shaped relationship with education; (3) the relationship between marriage rates and education is hump-shaped for women and increasing for men. We estimate that 2.5% of American women were childless because of poverty and 8.1% were childless because of a high opportunity cost of childrearing. Over time, we show that historical trends in total factor productivity and in education led to a U-shaped response in childlessness rates while fertility of mothers decreased.

Keywords: Fertility, Childlessness, Marriage, Education, Structural Estimation.
JEL Classification Numbers: J11; O11; O40.

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1 Introduction

Childlessness is an overlooked reality that cannot be explained by the current economic studies on fertility. At least three arguments show how important it is to distinguish between the extensive margin (1− childlessness rate) and the intensive margin (fertility of mothers) of fertility. The first follows from a historical analysis of the evolution of both margins. The left panel of Figure 1 shows that from one generation to another, childlessness and the completed fertility of mothers can either be negatively or positively correlated, and that in the long run the relationship is surprisingly positive.\(^1\) Second, the right panel of Figure 1 shows the relationship between childlessness and the completed fertility of mothers across education levels for a given cohort of American women (the sample is detailed in Section 2). We see that there exists an education threshold \(e\) below which childlessness and fertility are positively correlated as education changes, while this correlation is negative above \(e\).

Figure 1: Completed Fertility of Mothers vs. Childlessness (married women). By Cohort (left panel) and by Education Levels (right panel) in the USA

Third, childlessness is key to understanding differences in average fertility across countries. Taking the international data of Sardon (2006) for the cohort born in 1955, the difference in completed fertility between each country and the U.S. can be decomposed into the extensive and intensive margins (Table 1). In some countries, the completed fertility is higher (lower) than it is in the U.S. because both margins are higher (lower). For other countries, only one margin explains cross-country differences in completed fertility. In Austria, Belgium and Denmark, motherhood rates are higher than in the U.S. but total fertility is lower because mothers have fewer children. In France and Sweden, on the contrary, the fertility of mothers is similar to U.S. levels, but motherhood rates are much higher. Hence, the driving force

\(^1\)A linear regression leads to a positive relationship between childlessness and the completed fertility of mothers over time.
behind fertility differentials between France or Sweden and the U.S. for the 1955 cohort is childlessness. This confirms that both margins often play in opposite directions and that studying childlessness is necessary to understand international differences in fertility.

To summarize, the relationship between childlessness and fertility is not uniformly negative over time, across education groups, or across countries (which would be the case if childlessness was nothing more than low fertility). This reveals that economic incentives do not always lead the two margins to respond in the same direction, as expected based on the previous literature on fertility decisions. As average fertility is composed of both margins, ignoring the specific determinants of childlessness can lead to unexpected consequences from a policy perspective. Hence, in this paper, we investigate what mechanisms make the extensive margin of fertility behave differently from the intensive margin, and how do economic changes affect these two margins.

To identify the relevant mechanisms behind variations in the two margins, we need more than the observation that fertility and childlessness could be positively correlated, as this could happen through a variety of channels. We therefore look at the education gradient of fertility, conditionally on fertility being positive, and of childlessness rates. We do so for both single and married persons, hence also considering marriage rates by education level. From U.S. Census Bureau data for the year 1990 we find that: (i) the fertility of mothers decreases with education, for both married and single women, (ii) there is a U-shaped relationship between childlessness and education both for single and married women, and (iii) the relationship between marriage rates and education is hump-shaped for women and increasing for men (see Section 2). To the best of our knowledge, the first two facts have not been documented for both single and married women before, and there is no theory which accounts simultaneously for these three facts.
Observations (i) and (ii) are strongly indicative of a fertility-education relationship in which fertility jumps from 0 to positive, and then declines. Understanding this discontinuity requires to posit a nonconvexity in the constraints faced by households, which is, in our case, a minimum consumption requirement to have children. The existence of high levels of childlessness among disadvantaged groups in the United States is described by McFalls (1979).\footnote{2}

He argues that lower-income groups are more exposed to subfecundity causes (venereal diseases, malnutrition, psychopathological problems, environmental factors) than the rest of the population. The poor also have less access to quality medical services, which makes them more vulnerable to medical mistakes in abortions and cannot afford to buy fertilization services. Consequently, poor individuals are more affected by subfecundity factors because they do not have access to the same technologies as wealthier individuals.\footnote{3}

Individuals may therefore remain childless for three reasons. First, “natural sterility”, can randomly affect any individual. Second, “social sterility”, prevents poor women who do not have the minimum amount of commodities needed to procreate. Third, educated women who are not naturally sterile can remain childless because bearing and rearing a child takes time, and this opportunity cost is high for them in terms of foregone labor income. The U-shaped relationship between childlessness and the mother’s education is driven by the coexistence of social sterility and of a high opportunity cost of parenting for educated women.

The model economy of this paper (Section 3) is composed of men and women who play a three-stage game. During the first stage, each individual is randomly matched with a partner of the opposite sex and decides whether to marry or not.\footnote{4} Second, they discover whether they are naturally sterile, at no cost. And third, couples and singles make decisions about consumption and, if they can, fertility. We assume a collective negotiation process (see Chiappori and Donni (2009) for a literature review) to model the behavior of households. As shown by Chiappori (1988), this framework has considerable empirical support.

Marriage entails costs and benefits. For men, it opens the possibility of having children. As a counterpart, some of their time will be allocated to child-rearing. For women, a husband alleviates the time cost of raising children. Marriage also generates economies of scale since spouses share the expenses of household public goods. Being in a relationship also allows

\footnote{2}{The negative relationship between social sterility and income has already been documented in Wolowyna (1977), for 1971 Canadian Census data. Romaniuk (1980) provides a good discussion of the existence of high levels of childlessness in very poor societies.}

\footnote{3}{Social sterility also covers situations in which living conditions are incompatible with raising a child. This includes poor, cramped or unsafe housing that could put a small child at risk. In such case, women could have children from a purely biological standpoint but do not try to because of poor living conditions. The absence of pre- and postnatal care could also play the same role.}

\footnote{4}{To simplify, we assume the match is done randomly and that there is no second round. Results with a positive degree of assortative matching are given in Appendix C.8.}
saving time, which is particularly precious for highly educated people. The hump shaped relationship between marriage and education (Fact (ii)) is related to the high childlessness rates of uneducated and highly educated single women. For a man, marrying a woman who is too poor and requires help from the husband to procreate is less attractive. Marrying a woman with low incentive to have children (high opportunity cost) is hard as highly educated women reject marriage offers more often.

For simplicity, we abstract from unexpected births and cohabitation. Unwanted births are important from a social and individual point of view, as they can sharply reduce individual well-being. Bumpass and Westoff (1970) as well as Mosher et al. (2012) show that unwanted births concern mainly least educated women. Excluding these women from our dataset would then reinforce the U-shaped pattern of childlessness with respect to education.\(^5\) Cohabitation covers two aspects: making decisions together, and/or receiving help from another person. Only the second aspect is taken into account in the model. For the period we consider, even though cohabitation was unlikely (Bumpass and Westoff (1970)), so was single motherhood. Some of the women who are treated as single might then have an unmarried partner. In the US of the 50’s and the 60’s, laws against cohabitation and pre-marital sex co-exist with laws against interracial marriage and no-fault divorce. Then, \textit{de facto}, some situations of cohabitation (such as interracial unions) and out of marriage births cannot be legitimized by a marriage at this time. 1990 Census data unable us to go deeper on this issue (see Appendix A.1 for details).

By abstracting from education choices, we implicitly take education as exogenous to marriage and fertility decisions. This simplification is based on the idea that education is decided first, but we acknowledge that in some cases causality could be reversed (see for example Rindfuss et al (1980)). For instance, women with a strong preference for children could choose to drop out school to marry and have children. Our contribution should be understood as providing a unified theory of fertility, childlessness and marriage in the US where we show the importance of education and income gradients. Alternative explanations remain possible and potentially relevant.\(^6\)

The structural parameters of the model are identified using the three facts enumerated at the beginning (Section 4). Average fertility decreases with mother’s education, due to the

\(^5\)Unwanted first births can reduce childlessness rates as some women would not have children at all if they were able to perfectly control their fertility. Using data from the National Fertility Study (NFS), Bumpass and Westoff (1970) estimate that only 5% of first births were reported as unwanted between 1960 and 1965. Using data from the National Survey of Family Growth (NSFG) in 2002, Mosher et al. (2012) estimate that this percentage is 8.5%.

\(^6\)If we were able to remove women having low education because they gave birth unexpectedly would however strengthen the U-shaped relationship between education and childlessness.
higher opportunity cost of rearing children for more educated women. However, we show that Malthusian mechanisms, where education positively affects fertility, can appear for part of the population. As this positive effect concerns few individuals in the U.S., we do not observe it on an aggregate level. Concerning the U-shaped relationship between childlessness and education, we estimate that 2.5% of American women are socially sterile and 8.1% are childless because of the high opportunity cost.

In a historical experiment (Section 5), we show that changes in education and in total factor productivity allow replicating the overall decreasing trend in fertility for the generations born between 1871 and 1964, while childlessness first decreased and then increased. We interpret the changes in childlessness first by a decline in childlessness created by poverty and, after the middle of the twentieth century, by an increase in childlessness created by a rise in the opportunity cost of having children.

To the best of our knowledge, this paper constitutes the first study of the determinants of marriage and fertility along its two margins in a unified framework. Gobbi (2011) studies the determinants and the evolution of marital childlessness during the twentieth century. Aaronson et al. (2011) focus on a quantity-quality approach and look at how the Rosenwald Rural Schools Initiative in the early twentieth century affected fertility along both the extensive and intensive margins. They show, in particular, that the expansion of schooling opportunities decreased the price of child quality, which decreased the proportion of women with the highest fertility rates as well as childlessness rates. Both contributions assume that childlessness is always a choice. We differ from these papers by looking at the role played by the three sources of childlessness and marriage opportunities.

A growing literature is concerned both with family composition and fertility choices. However, it does not allow the three facts to all be explained together. Greenwood et al. (2003) and Regalia et al. (2011) analyze both marriage and fertility decisions in a dynamic programming framework where individuals can divorce. Instead of increasing the complexity of their set-up further to allow for different motives for childlessness, we develop a model abstracting from divorce and concentrating on the mechanisms behind fertility decisions.

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7 Using older data from US census, Jones and Tertilt (2008) show that it was observable for cohorts born in 1828, 1848 and 1868.
8 With other data sources, alternative estimations of childlessness can be found in the literature. Using the National Survey of Family Growth for the year 1982, Mosher (1985) defines as “impaired fecund” women who are non-surgically sterile, for whom it is difficult or dangerous to have a baby, or who had a three-year or longer interval of formal or informal marriage and non use of contraception without a pregnancy. He estimates that 8.4% of American women 15-44 years of age were in such case.
9 With close arguments and through a cross-country analysis, Poston and Trent (1982) show the existence of a U-shaped relationship between the development level of a country and its childlessness rate.
2 Three Facts from 1990 US Census

We use the 5\% sample of the U.S. Census 1990, taken from the Integrated Public Use Microdata Series (IPUMS), and restrict our attention to the “ever-married, spouse present” and “never-married” women having completed their life-cycle fertility.\(^{10}\) We look at women aged between 45 and 70 years old.\(^{11}\) We divide the female population into 12 categories of education and report the theoretical average number of years of education, \(e\), for each category as well as the unweighted number observations (sum of singles and ever-married) per category. Table 2 summarizes the three stylized facts we focus on, computed using the weighted observations: completed fertility of mothers, childlessness rates of women and marriage rates of men and women for twelve education levels.

**Fact 1:** Fertility of mothers decreases with education, for both married and single women.

This fact is well known for married women. It is less known that, fertility of single women (conditionally on being mothers) also decreases with education. The negative relation between fertility and education has already been stressed in many papers without conditioning on both marital and motherhood status (see Becker (1981) pages 150-151, de la Croix and Doepke (2003) and Jones and Tertilt (2008)).\(^{12}\)

**Fact 2:** Childlessness exhibits a U-shaped relationship with education for both single and married women.

The relationship between childlessness and education is not monotonic unlike the relationship between fertility and education. As explained in the Introduction, this U-shaped relationship is explained by the existence of both poverty and opportunity cost reasons leading to childlessness. For married women, the U-shaped relationship looks more J-shaped because marriage is used as a way of insuring against social sterility for women with the lowest levels of education. As shown in the right panel of Figure 1, there exists an education threshold for which childlessness is minimal (Grade 11 for both singles and married). After this education threshold, childlessness increases with education as highly educated women have a higher

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\(^{10}\)The 1990 census is the last one for the U.S. to report completed fertility for women that are older than 44 years old. We drop from our sample women who are separated, divorced, widowed and married when their spouse is absent. The downwards relationship between fertility of mothers and education and the U-shaped relationship between childlessness and education hold for these women as well. These categories account for 30.5\% of women. We exclude them from the sample because we do not know since when they are no longer with their partner.

\(^{11}\)The oldest and the youngest women of the sample have decided to marry and to become mothers in somewhat different social and economic conditions. As shown in Appendix A.3, the facts presented in this section hold for each five-year cohort.

\(^{12}\)See Appendix A.2 for more details on the single mothers with little education.
### Table 2: Facts from U.S. Census 1990

<table>
<thead>
<tr>
<th>Nb</th>
<th>Education Level</th>
<th>e</th>
<th>N. obs.</th>
<th>Childlessness Rates</th>
<th>Completed Fertility of Mothers</th>
<th>Marriage Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Married</td>
<td>Single</td>
<td>Married</td>
</tr>
<tr>
<td>1</td>
<td>No school</td>
<td>0</td>
<td>12122</td>
<td>0.088</td>
<td>0.755</td>
<td>4.880</td>
</tr>
<tr>
<td>2</td>
<td>Grade 1-4</td>
<td>3</td>
<td>14050</td>
<td>0.074</td>
<td>0.590</td>
<td>4.791</td>
</tr>
<tr>
<td>3</td>
<td>Grade 5-8</td>
<td>7</td>
<td>84243</td>
<td>0.078</td>
<td>0.631</td>
<td>3.916</td>
</tr>
<tr>
<td>4</td>
<td>Grade 9</td>
<td>9</td>
<td>38121</td>
<td>0.072</td>
<td>0.560</td>
<td>3.647</td>
</tr>
<tr>
<td>5</td>
<td>Grade 10</td>
<td>10</td>
<td>57213</td>
<td>0.066</td>
<td>0.588</td>
<td>3.519</td>
</tr>
<tr>
<td>6</td>
<td>Grade 11</td>
<td>11</td>
<td>49413</td>
<td>0.059</td>
<td>0.524</td>
<td>3.485</td>
</tr>
<tr>
<td>7</td>
<td>Grade 12</td>
<td>12</td>
<td>479703</td>
<td>0.076</td>
<td>0.781</td>
<td>3.079</td>
</tr>
<tr>
<td>8</td>
<td>1 year of college</td>
<td>13</td>
<td>178274</td>
<td>0.083</td>
<td>0.839</td>
<td>2.961</td>
</tr>
<tr>
<td>9</td>
<td>2 years of college</td>
<td>14</td>
<td>53428</td>
<td>0.077</td>
<td>0.825</td>
<td>2.976</td>
</tr>
<tr>
<td>10</td>
<td>Bachelor degree</td>
<td>16</td>
<td>99046</td>
<td>0.101</td>
<td>0.934</td>
<td>2.788</td>
</tr>
<tr>
<td>11</td>
<td>Master degree</td>
<td>17</td>
<td>56855</td>
<td>0.137</td>
<td>0.959</td>
<td>2.606</td>
</tr>
<tr>
<td>12</td>
<td>Doctoral degree</td>
<td>20</td>
<td>4612</td>
<td>0.191</td>
<td>0.957</td>
<td>2.408</td>
</tr>
<tr>
<td>all</td>
<td></td>
<td></td>
<td>1127080</td>
<td>0.081</td>
<td>0.787</td>
<td>3.160</td>
</tr>
</tbody>
</table>

Note: Standard errors are given in Appendix A.4. Marriage rates of men are weighted (multiplied by 0.998, the ratio between the total number of married women and the total number of married men) in order to have an equal number of men and women in the data, as in the model. Each observation has a weight given by the Census and represents between 2 and 186 individuals. These weights are used to compute the facts.

Fact 3: There is a hump-shaped relationship between marriage rates and education levels for women. Marriage rates (weakly) increase with education for men.

Marriage rates are the highest for intermediate levels of education: from Grade 5 to Bachelor degree, marriage rates are above 90%. For women, these rates are lower for extreme levels of education: less than 70% for women with no education and 75% for women having a PhD. Marriage rates are low for lowly educated men (72% for those without any education) but remain high for the most educated.

Appendix A.5.1 shows that the three facts are true for Whites, Blacks, Natives, Asians and Hispanics separately. Appendix A.5.2 shows that the facts are also robust if we remove foreign born individuals from the sample, and hence do no rely on the behavior of foreigners.

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13 For single women, the U-shaped pattern of childlessness with respect to education is robust if we merge the first three groups (“no school”, “grade 1-4” and “grade 5-8”). Single women who are in the “no school” or “grade 5-8” group have an average childlessness rate that is significantly higher from the average rate of those in “grade 10”. Childlessness rates of single women in all education groups are significantly different from that in “grade 11”, except for those in “grade 9”.
who have made their marriage and fertility decisions in another country.

3 Theory

We consider an economy populated by heterogeneous adults, each being characterized by a triplet: sex $i = \{m, f\}$, wage $w_i$, and non-labor income $a_i$. Marriage is a three stage game. First, people enter the marriage market ignoring their sterility status and decide whether or not to marry the partner they have been randomly matched with; second, they discover their sterility status at no cost; and third, they decide how much to consume and, eventually, how many children to have, if any. The utility of an individual of sex $i$ is

$$u(c^i, n) = \ln(c^i) + \ln(n + \nu),$$ (1)

where $c^i$ is the individual’s consumption and $n$ the number of children that he or she has. $\nu > 0$ is a preference parameter that allows for the existence of childlessness, as the individual utility remains defined when $n = 0$. In line with Iyigun and Walsh (2007), children are considered a public good for the couple and there are no gender differences in preferences. Assuming homogeneity in preferences, both across and within genders, is a way to measure by how much economic incentives account for our three stylized facts without relying on differences in preferences. Differences in fertility and childlessness result from the structure of the marriage market and the heterogeneity on labor and non-labor incomes.

Time endowment is to be shared between working and child rearing. This time equals 1 for a married person, and $1 - \delta^i$ for a single of sex $i$. This implies that marriage entails economies of scale in time (for instance, sharing domestic activities). Allowing for gender differences in the time endowment of singles does not only mean that women (or men) might be more productive in doing domestic tasks but also that there might be somebody else to help, in particular in the case of a single mother (a mother, a sister, a father, or a lover).

We assume that single women can have children whereas single men cannot. Having children entails time costs. First, there is a fixed cost, $\eta \in (0, 1)$ to becoming a parent. This is justified by the fact that the first child costs more in terms of time than the following children.\footnote{Turchi (1975) estimates that the mean number of hours spent per day on childrearing for a one-child family is 1.4, for the first 18 years. For a two-child family it is 0.99 per child. And for a three-child family, it is 0.93. See Table 6.4 in Folbre (2008) for similar results.} In addition, there is a variable cost: each child needs $\phi \in [0, 1]$ units of time to be raised. When married, the husband bears an exogenous part $(1 - \alpha) < 1/2$ of the childrearing time, as in Echevarria and Merlo (1999). If single, the mother has to bear the full time-cost alone. This
makes single women more likely to be childless than married women.

To account for a positive degree of sterility among men and women that is uncorrelated to wealth, we assume that natural sterility is uniformly distributed over education categories. \( \chi \in [0,1] \) and \( \zeta \in [0,1] \) respectively describe the percentage of female and male who are naturally sterile. To model social sterility, we assume that in order to be able to give birth, a woman has to consume at least \( \hat{c} \):

\[
 c^f < \hat{c} \Rightarrow n = 0. \tag{2}
\]

This assumption has been amply justified in the Introduction by the fact that lower-income groups are more exposed to causes of subfecundity than the rest of the population. Notice that, unlike a goods cost of children, \( \hat{c} \) does not depend on the number of children that the mother will bear. A proportional goods cost does not imply social sterility, as the mother could choose a low enough number of children that is compatible with her budget constraint.

Each adult draws a non-labor income \( a^i > 0 \) from a distribution \( F^i(\bar{m}^i, \sigma^i_a) \), independent of his/her education.\(^{15}\) Non-labor income corresponds to the part of income that is uncorrelated with education.\(^{16}\) The total non-labor income for a couple equals \( a^f + a^m \). Wages per unit of time of men and women are respectively denoted \( w^f \) and \( w^m \). Wages depend on education and gender only. Each household has to pay a goods cost, \( \mu \), which is a public good within the household. This type of cost is commonly assumed in the literature and gives some incentive to form couples (see Greenwood et al. (2012)). Given these assumptions, the budget constraints, denoted as \( b^m(c^m) \) for a single man, \( b^f(c^f, n) \) for a single woman and \( b(c^f, c^m, n) \) for a couple, are as follows:

\[
 b^m(c^m) = c^m - (1 - \delta^m)w^m - a^m + \mu \leq 0 \quad \text{and} \\
 b^f(c^f, n) = c^f + \phi (1 + \eta(n)) w^f n - (1 - \delta^f)w^f - a^f + \mu \leq 0, \\
 b(c^f, c^m, n) = c^f + c^m + \phi (1 + \eta(n)) (\alpha w^f + (1 - \alpha)w^m) n - w^m - w^f - a^f - a^m + \mu \leq 0.
\]

with the fixed time cost being given by

\[
 \eta(0) = 0 \quad \text{and} \quad \eta(n) = \frac{\eta}{n} \text{ if } n > 0. \tag{3}
\]

The maximum number of children a single woman can have, given hertime constraint \( \phi(1 +

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\(^{15}\)We relax the independence assumption in Appendix C.3. Allowing for a non zero correlation between education and non-labor income is not essential for the results.

\(^{16}\)As shown in Jones et al. (2010), because of our log specification of the utility function, we need the non-labor income to be positive to generate the negative fertility-income relationship at the aggregate level. As mentioned by them, this could be gifts, lottery income or bequests.
η/n)n ≤ 1 − δ^f, is
\[ n_M = \frac{1 - \delta^f - \phi \eta}{\phi}. \]

The maximum fertility rate of a married woman equals
\[ \bar{n}_M = 1 - \alpha \phi \eta / \alpha \phi. \]

This is greater than the maximum fertility of a single woman (\bar{n}_M > n_M) because husbands help in the raising of children.

To model couples’ decision making, we assume a cooperative collective decision model.\(^{17}\) Spouses negotiate on \(c^m, c^f\) and \(n\). Their objective function is
\[ U(c^f, c^m, n) = \theta(w^f, w^m) u(c^f, n) + (1 - \theta(w^f, w^m)) u(c^m, n), \]
where \(\theta(w^f, w^m)\) is the wife’s bargaining power that depends on education and is given by
\[ \theta(w^f, w^m) \equiv \frac{1}{2} \theta + (1 - \theta) \frac{w^f}{w^f + w^m}. \]

We specifically assume that the negotiation power of spouses is bounded from below, with a lower bound equal to \(\theta/2\), and positively related to their relative wage. As education will be used as a proxy for potential wages, negotiation power is in fact positively related to the relative education of the spouse.\(^{18}\) The boundedness of the bargaining power function comes from the legal aspect of marriage: spouses have to respect a minimal level of solidarity inside marriage.

The different assumptions we have introduced imply some advantages to being married. It allows the cost of the household \(\mu\) to be shared. It increases the time endowment of the participants by allowing to share domestic activities. Being married is the only way for men to have children. It allows women to reduce the opportunity cost of children (as long as \(\alpha < 1\)). Marriage gives at least one of the partners the opportunity to increase his/her consumption compared to the situation where he or she remains single.

\(^{17}\)The fixed cost \(\mu\) ensures that there is always a surplus coming from marriage. By adopting a collective cooperative decision model, rather than a Nash bargaining process where potential spouses share the marriage surplus, we avoid marriage rates being equal to one. With a Nash bargaining process, and no frictions in the marriage market, the only way to allow for some proportion of singles would be to assume some negative shocks on the quality of the matching.

\(^{18}\)See de la Croix and Vander Donckt (2010) for a discussion. An alternative consists in assuming a negotiation power that depends on the spouses’ relative labor income rather than their relative education as in Iyigun and Walsh (2007). As suggested by Pollak (2005), we have also tried a specification that included non labor incomes in the bargaining power, results are provided in Appendix C.6.
In order to write down the conditions under which a marriage offer from a randomly drawn person of the opposite sex is accepted, one needs to define the value functions of being ex post in one of the following situations:

\[
\begin{align*}
V^{s,m} &\equiv \{\max u(c^m, 0) \text{ such that } b^m(c^m) \leq 0\} \quad \text{[single male]} \\
V^{s,f} &\equiv \{\max u(c^f, n) \text{ such that } b^f(c^f, n) \leq 0, 0 \leq n \leq \bar{n}_M, (2)\} \quad \text{[single female]} \\
\tilde{V}^{s,f} &\equiv \{\max u(c^f, 0) \text{ such that } b^f(c^f, 0) \leq 0\} \quad \text{[single sterile female]} \\
V^{\omega,i} &\equiv u(c^i, n) \text{ where } \{c^f, c^m, n\} = \arg \max U(c^f, c^m, n) \text{ such that } b(c^f, c^m, n) \leq 0, 0 \leq n \leq \bar{n}_M, (2) \quad \text{[married]} \\
\tilde{V}^{\omega,i} &\equiv u(c^i, 0) \text{ where } \{c^f, c^m\} = \arg \max U(c^f, c^m, 0) \text{ such that } b(c^f, c^m, 0) \leq 0 \quad \text{[sterile married]}
\end{align*}
\]

We now consider the decision to accept the marriage. Once a potential partner has been drawn, we know the vector \((w^f, a^f, w^m, a^m)\). This allows each individual to compare his/her expected utility as a single and as a married person and decide whether to accept marriage or not. Marriage occurs if:

\[
(\chi + (1 - \chi)\zeta)\tilde{V}^{\omega,f} + (1 - \chi)(1 - \zeta)V^{\omega,f} \geq \chi\tilde{V}^{s,f} + (1 - \chi)V^{s,f},
\]

\[
(\chi + (1 - \chi)\zeta)\tilde{V}^{\omega,m} + (1 - \chi)(1 - \zeta)V^{\omega,m} \geq V^{s,m}.
\]

We assume here that single women are not concerned with male sterility as they can have multiple sexual partners.\(^{19}\) Married women are more concerned with natural sterility as they can be matched with a sterile partner.

The solutions to the maximization problems are provided in Appendix B. Depending on which constraint binds, individuals can be in six different situations: [N] natural sterility, [S] social sterility when \(c^f < \hat{c}\) and \(n = 0\), [M] maximum fertility when the time constraint binds, [C] constrained fertility when \(c^f = \hat{c}\) binds and \(n > 0\), [O] opportunity cost driven childlessness when the constraint \(n \geq 0\) binds, and finally, [U] unconstrained fertility.

We here describe how the fertility of a married woman depends on her education and the couple’s non-labor income, keeping the husband’s education fixed. A similar logic applies for a single woman (Appendix B.2).

\(^{19}\)A woman who has \(n\) different sexual partners in her life meets only infertile partners with a probability \(\zeta^n\). If natural sterility among males is 5\% and a woman has only two different partners in her lifetime, she has a probability of 0.0025 of meeting only infertile partners. According to the National Survey of Family Growth (2002), the average number of lifetime sex partners for women who have always been single in the U.S. is 7.44.
Figure 2: Fertility Conditional on Being Married when $a \in [A_1, A_2]$ and when $a > A_5$

Abstracting from natural sterility $[N]$, Figure 2 shows fertility decisions when the couple’s non-labor income is low (left) and high (right). All the non-labor income thresholds $A_1$, $A_2$, $A_5$, and the wage thresholds $W_{D_f}$, $W_{H_f}$, $W_{A_f}$, $W_{C_f}$, $W_{F_f}$, $W_{H_f}$ are defined in Appendix B.3. We see that different non-labor incomes drive different fertility behaviors when female’s wage is low. A woman with a very low wage ($w_f < W_{A_f}$) in a marriage where the non-labor income is low ($a \in [A_1, A_2]$) cannot reach the minimal level of consumption that enables her to procreate. Hence, she is socially sterile $[S]$. When her education increases ($W_{A_f} \leq w_f < W_{C_f}$), procreation is feasible but not optimal. For $W_{C_f} \leq w_f < W_{A_f}$ fertility increases with female wage. In this Malthusian case $[C]$, the minimal consumption constraint is binding with equality and an increase in female wages can affect fertility positively through an income effect. The Malthusian segment disappears as wealth increases. With a high non-labor income ($a > A_5$), women can always consume more than $\hat{c}$ and social sterility does not occur for any value of the female wage. A woman with a low wage has a low opportunity cost to childrear and it is optimal for the couple to have the maximal number of children $[M]$. An increase in the female wage implies a larger opportunity cost of childrearing in terms of foregone income and hence fertility decreases $[U]$, up to the point when it is optimal to be childless $[O]$.

According to Figure 2, a rise in non-labor income can reverse the relationship between education and fertility. This is consistent with Skirbekk (2008), who shows a general shift during the nineteenth century in Europe and North America from a positive to a negative relation between social status and fertility, and with Vogl (2014) according to whom, in developing countries, children from larger families had richer parents before 1960 and the opposite since then. Table 2 does not display the Malthusian positive relationship between fertility and education (left panel of Figure 2) as, nowadays, few American women have
children while being in the constrained fertility case [C]. In Section 4 we estimate that 1.8% are in such case. In order to see Malthusian mechanisms in aggregate US data, we need to look at historical data, in which more women face fertility constraints. US Census data display such a positive relationship for lowly educated women born before 1880 (see Jones and Tertilt (2008) Table A4).

4 Identification of Parameters and Simulations

The model is now fit to match the facts provided in Table 2. There are in total fifteen parameters. We first set a priori four of these parameters (wage equation and natural sterility), based on empirical information. The eleven remaining parameters are estimated using a minimum distance estimation procedure.

4.1 A priori information

Wages are computed as follows:

\[ w_e = \gamma z \exp\{\rho e\} \]  

where \( e \) denotes the average number of years of education in each category (Table 2). The values of the gender wage gap \( \gamma \) and of the Mincerian rate of return to schooling \( \rho \) are estimated using Census data (see Appendix A.7.4 for details). The results lead to setting \( \gamma \) equal to 0.869 and \( \rho \) equal to 0.092.\(^{20}\) \( z \) is a normalization factor that will be used in Section 5.1 to capture the trend in total factor productivity.

Normalizing wages with respect to the wage of a man who has a doctoral degree, the minimum wage equals 0.158 for men and 0.137 for women. The main drawback of the Mincerian approach is to assume the same return to schooling for all schooling levels. In Appendix A.8, we discuss the importance of this assumption for our results. The asset of the Mincerian approach is to let income depend on two parameters only, \( \gamma \) and \( \rho \), which can be the subject of counterfactual experiments. Such counterfactual exercises are provided in Subsection 5.2.

The two sterility parameters, \( \chi \) and \( \zeta \), are fixed using information from the literature on natural sterility. The ideal population to measure sterility among couples is one where marriage is associated with the desire to have children and where women marry young, do

\(^{20}\)We assume that the gender wage gap \( \gamma \) is the same across education categories. Assuming that \( \gamma \) increases with education, as some studies in the 1980’s suggested, modifies the results only marginally. Lips (2003) shows that this “education penalty” almost disappeared during the 1990’s. The estimated value for the gender wage gap is similar to the estimates of Erosa et al. (2010). The estimate of \( \rho \) is also in line with the literature (Krueger and Mikael (2001)).
not divorce (e.g. because of sterility), are faithful to their husband and live in a healthy environment. The closest to this ideal are the Hutterites. According to Tietze (1957), who studies sterility rates among this population, we should set the percentage of naturally sterile couples, \( \chi + (1 - \chi)\zeta \), at 2.4%. Two alternative measures of sterility are discussed in Appendix C.2. Our reading of the literature on the prevalence of fertility problems is that roughly half of them with a diagnosed cause are related to the man, and half to the woman.\(^{21}\) We therefore set \( \chi \) equal to \( \zeta \). The two restrictions lead to \( \chi = \zeta = 1.21\% \).

4.2 Minimum Distance Estimates

The eleven remaining parameters, listed in Table 3, are identified by minimizing the distance between 72 empirical and simulated moments. These moments are the completed fertility of mothers and childlessness rates among both singles and married women, and the marriage rates for women and men, for the 12 education categories listed in Table 2. The objective function to minimize is given by:

\[
    f(p) = (d - s(p))'[W](d - s(p))'
\]

where \( p \) is the vector of the parameters of the model, \( d \) denotes the vector of the empirical moments and \( s \) the vector of the simulated moments, depending on the parameters. \( W \) is a weighting matrix with \( 1/d^2 \) on the diagonal, and zero elsewhere.\(^{22}\)

To compute the vector of the simulated moments, we proceed as follows. We draw 100,000 women for each category of education. Each woman draws a potential husband from the empirical distribution of education levels among men. Each individual (man and woman) also draws a non-labor income. We assume that the non-labor income follows a log-normal distribution of mean \( \kappa_a \) and variance \( \sigma_a^2 \). Writing \( \kappa_a = \ln(\bar{m}_a\bar{w}) - \frac{\sigma_a^2}{2} \) where \( \bar{w} \) denotes women’s average wage, the parameter \( \bar{m}_a \) can be interpreted as the average ratio of non-labor income to labor income. Then, for each category of education for women, we obtain 100,000 decisions about marriage and fertility that we can average to calculate the simulated moments.

\(^{21}\)This estimation is quite imprecise as definitions of sterility and infertility used in the literature lack uniformity (see Gurunath et al. (2011)).

\(^{22}\)This matrix is often used in the literature instead of the optimal weighting matrix, i.e. the inverse of the variance-covariance matrix of the empirical moments (Duffie and Singleton (1993)). When using the optimal weighting the moments computed from a larger number of observations have a lower standard errors and a higher weight. Consequently, for these moments, a higher distance between the data and the simulated moments is more heavily penalized in the objective function. In our case, using the optimal matrix lowers the weight of the observations for extreme education levels, which are the most useful for identification.
We then minimize the objective function $f(p)$ (see implementation notes in Appendix C.1). In the numerical implementation, we also assume that the number of births is an integer rather than a continuous variable. It does not alter the main mechanisms at play but simplifies computations (and is also realistic).

### 4.3 Results

The identified parameters are listed in Table 3. Non-labor income amounts on average to 100% of labor income. This number seems quite high, unless we interpret the non-labor income as including, gifts, bequests, capital income and transfers (including social security) that are not correlated with the education level of the recipient. To have an idea of the magnitude of $\sigma_a$, we computed a Gini coefficient on women’s simulated life-cycle income, $w^f + a^f$, which came 0.16. The estimated $\sigma_a$ is relatively on the low side, but this is not surprising as some dimensions of inequality are absent from the model, such as wage dispersion for similar education levels.

To interpret the value of $\hat{c}$ and $\mu$, remember that wages vary on a scale from 0.137 to 0.869 for women. For an average couple where both partners have 12 years of education and 2.5 children, the household cost represents 18.0% of consumption. This number is in line with the estimates found in Greenwood et al. (2012). A single woman with the lowest wage will

---

**Table 3: Identified Parameters**

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance of the log-normal distribution</td>
<td>$\sigma_a$</td>
<td>0.247</td>
<td>0.012</td>
</tr>
<tr>
<td>Average ratio of non-labor income to labor income</td>
<td>$\bar{m}_a$</td>
<td>1.001</td>
<td>0.012</td>
</tr>
<tr>
<td>Preference parameter</td>
<td>$\nu$</td>
<td>9.362</td>
<td>0.146</td>
</tr>
<tr>
<td>Minimum consumption level to be able to procreate</td>
<td>$\hat{c}$</td>
<td>0.399</td>
<td>0.009</td>
</tr>
<tr>
<td>Good cost to be supported by a household</td>
<td>$\mu$</td>
<td>0.272</td>
<td>0.013</td>
</tr>
<tr>
<td>Bargaining parameter</td>
<td>$\theta$</td>
<td>0.864</td>
<td>0.014</td>
</tr>
<tr>
<td>Fraction of childrearing to be supported by women</td>
<td>$\alpha$</td>
<td>0.524</td>
<td>0.005</td>
</tr>
<tr>
<td>Time cost of having children</td>
<td>$\phi$</td>
<td>0.206</td>
<td>0.003</td>
</tr>
<tr>
<td>Fixed cost of children</td>
<td>$\eta$</td>
<td>0.114</td>
<td>0.006</td>
</tr>
<tr>
<td>Time cost of being single (men)</td>
<td>$\delta^m$</td>
<td>0.256</td>
<td>0.015</td>
</tr>
<tr>
<td>Time cost of being single (women)</td>
<td>$\delta^f$</td>
<td>0.077</td>
<td>0.013</td>
</tr>
</tbody>
</table>

23 Appendix C.4 explains how the standard errors are computed. Appendix C.5 provides the identified parameters under the assumption that they are race specific, and that marriage markets are segmented by race. Appendix C.5 also gives the estimated parameters after having removed from the sample the “disables”. A discussion of whether one should include them or not in the sample is provided in Appendix A.6. Appendix C.5 also breaks down childlessness into its components for each subgroup.
need a non-labor income higher than 0.671 \((\hat{c} + \mu, \text{see Appendix B.2})\) not to be socially sterile.

The minimum negotiation power of a spouse is \(\theta/2 = 0.432\). Alternative specifications for the bargaining power function \(\theta\) have been tested. Appendix C.6 shows that fixing the bargaining power to 1/2 or making it depend on relative potential incomes (hence including non labor incomes \(a^f\) and \(a^m\)) does not improve the matching with the data.

Spouses share childrearing time. We estimate that men are involved for 47.6\% of this time. The interpretation of \(\alpha\) should be broader than only active childcare spent by parents. Quoting Folbre (2008), page 117: “Parents do more than merely devote time to their children. They work at the task of providing both active and passive care”. Activities such as looking for care centers or schools, or cleaning the car, in which the parent is not directly with the child also enter the amount of time parents devote to their children. These activities are included in \(\alpha\) but are hard to estimate from surveys or time diaries. The values for \(\phi\) and \(\eta\) imply that the first child costs \(\phi(1 + \eta) = 22.9\%\) of the time endowment of one parent, while the second child costs 20.6\%. Following the values of \(\alpha\), \(\eta\) and \(\phi\), the maximum number of children that a married woman can raise is nine, while a single woman can have four children at most.

Our estimates of the \(\delta^f\) and \(\delta^m\) are such that \(\delta^f < \delta^m\), implying that single women lose less time than single men because of their singleness. This can reflect the fact that women are more efficient at house keeping than men, or that single women find help more easily from others than single men. If we believe that singles have periods during which they cohabit with another person, the lower \(\delta^f\) may also reflect that women benefit more from cohabitation than men. Notice finally that a positive \(\delta^m\) is in line with the literature showing that married men are more productive (e.g. better health, higher wages) than single men (Korenman and Neumark (1991)).

The simulated moments for the intensive and extensive margins of fertility that we obtain are represented in Figures 3 and 4. We reproduce the completed fertility pattern of single and married mothers (Fact 1), and the U-shaped relationship between childlessness and education for both married and single women (Fact 2). As for the marriage rates of men and women, Figure 5 shows that the model reproduces the hump-shaped relationship between marriage rates and education levels for women, and the positive correlation between marriage and education for men (Fact 3). For highly educated men, the high proportion of time they have to spend in rearing children (48\%) implies that having children and being married is costly for them.\(^{24}\) However, the 26\% gain of time endowment by being married (\(\delta^m\)) is enough to

\(^{24}\) Setting \(\alpha = 1\) increases the marriage rates of highly educated men but the marriage rates of highly
make them marry. The only weak aspect of the fit concerns the low education groups, for whom the theory overestimate the marriage rates by 10 to 18 percentage points.

As a test of the theory, we compare our results to empirical observations that were not used to identify the parameters. Looking at the fertility of married husbands is particularly interesting as it should allow us to assess whether our estimation of \( \alpha \) is appropriate. Indeed, the slope of the relationship between the husband wage and fertility is determined by \( \alpha \) (Equation (7) in Appendix B). Figure 6 reports the results. The theory can reproduce the negative relationship between average fertility and the education of husbands; the estimate of its slope is very good. This indicates that our estimation of the opportunity cost for men of having children is adequately measured. Figure 6 also reports the childlessness rate of married men with respect to their education. The model also captures the U-shaped relationship between both variables.

For parsimony we have included only one dimension of heterogeneity. Beyond non-labor income, preferences are the obvious candidate for another source of heterogeneity: some people might care more about having many children than others. We accordingly introduced one new parameter: the variance of the (lognormal) distribution of \( \nu \) across individuals. Estimating this parameter together with the other parameters leads to the result that this variance should be zero. Introducing variability in \( \nu \) increases variability of fertility when it is unconstrained, but this does not help to match the moments better. Hence heterogeneity in preferences on top of heterogeneity in non-labor income does not bring any advantage for our analysis.

Figure 7 (left) provides the proportion of women in each fertility situation, with respect to their education. Uneducated women are either socially sterile \([S]\), in the constrained fertility \([C]\), or with the maximum number of children \([M]\). We estimate that 1.8% of American women are in this situation. This means that although aggregate fertility data suggests that Malthusian checks no longer prevail and Becker’s model describes the negative relationship between education and fertility well, our model detects that some categories of the population are still affected by Malthusian mechanisms. Maximum fertility regimes, where poorly educated women would like to have more children but are constrained by their time endowment, concern 2.0% of American women (0.7% singles and 1.4% married). Women with the highest education are either childless \([O]\) due to the high opportunity cost to childrear, or mothers in the unconstrained fertility case \([U]\).

To dig deeper into the roots of childlessness when single, we have computed the fraction of educated women are now strongly underestimated as they lose their incentive to marry and have children (we discuss this in more detail in Appendix C.7).
Figure 3: Childlessness Rate and Completed Fertility of Mothers, by Years of Schooling, Married Women. Data (black), Simulation (grey), Education Categories (labels)

Figure 4: Childlessness Rate and Completed Fertility of Mothers, by Years of Schooling, Single Women. Data (black), Simulation (grey), Education Categories (labels)

Figure 5: Marriage Rates of Women and Men, by Years of Schooling. Data (black), Simulation (grey), Education Categories (labels)
completed fertility
years of schooling

childlessness rate
years of schooling

Women’s Education
\[
\begin{array}{lcccc}
\% & 1-2 & 3-9 & 10-12 & \text{Total} \\
[N] & 2.3 & 2.3 & 2.3 & 2.3 \\
[S] & 12.1 & 2.5 & 0.6 & 2.5 \\
[M] & 10.4 & 2.1 & 0.3 & 2.0 \\
[C] & 26.0 & 1.5 & 0.0 & 1.8 \\
[O] & 0.6 & 6.3 & 19.4 & 8.1 \\
[U] & 48.6 & 85.3 & 77.4 & 83.3 \\
\end{array}
\]

Figure 6: Completed Fertility of Married Fathers (left), Childlessness rates of Married Men (right), by Years of Schooling. Data (black), Simulation (grey), Education Categories (labels).

Figure 7: Regimes (left) and Childlessness Causes (right) as Function of Women’s Education.

single women in [O], whose marriage offer was rejected. This proportion is around 0.06%. Hence, the theory suggests that the majority of childless single women who are neither socially nor naturally sterile are single because they have rejected a marriage offer.

Figure 7 (right) plots the percentage of women that are childless in each category of education, distinguishing whether it is driven by the opportunity cost of raising children, by poverty or by biological reasons. Considering all education categories, social sterility concerns 2.5% of American women (2.1% singles and 0.4% married), while childlessness driven by a high opportunity cost concerns 8.1% of American women (3.3% singles and 4.8% married). Childlessness concerns essentially either lowly educated (socially sterile) or highly educated women, for both married and single people. The percentage of socially sterile women decreases with education, while the percentage of women who are childless because the opportunity cost of childrearing increases with education. This explains the U-shaped
relationship between education and childlessness (Fact 2).

The hump shaped relationship between marriage and education is related to the high childlessness rates of uneducated and highly educated single women as having children is the main motivation of men to marry in our set-up. Especially marrying a woman who is very poor and requires help from the husband to procreate is less attractive than marrying better educated women.

Despite having assumed random matching in the marriage market, the model predicts that individuals are more likely to marry someone with a similar level of education. This level of assortativeness is lower in the simulations than in the data because of the static nature of the model and the assumption that life only brings one chance to get married. In Appendix C.8, we provide a way to measure the degree of assortativeness in the data and the model and discuss this question further.

5 Counterfactual Experiments

Now, we use the parameters identified in Section 4 in several counterfactual experiments. The first is a historical experiment to understand the changes in childlessness and fertility for the cohorts born between 1871 and 1964. Then, we look at how an increase in inequality and a change in the wage gap affect childlessness and fertility.

5.1 Historical Experiment

We have already stressed in the Introduction that childlessness and fertility are not always negatively correlated over time (left panel of Figure 1, for married women). Apart from the baby boom episode, the secular pattern of fertility is monotonically decreasing. Childlessness too, over the long-run, declined, from around 16% in the end of the nineteenth century to around 12% for the last cohorts for which completed fertility can be observed. All historical data are available in Appendix A.7. In this section we use our theory to shed light on these long term trends.

In a cross-section of households, we have seen (Figure 7) that rising education leads to a decrease in childlessness for low levels of education and an increase in childlessness for high levels of education. The question we are investigating now is whether the same reasoning could be applied to explain the secular pattern of childlessness and fertility. To do so, we will rely on two mechanisms: the rise in education as observed in the different censuses, and an overall trend in income, related to the macroeconomic trend in total factor productivity.
Before turning to the results of the simulations, let us make clear that there are two features of the data that we have no hope to capture. The first is the baby boom, for the cohorts born between 1911 to 1940. The mechanisms that could generate the baby boom are absent from our framework.\textsuperscript{25} The second particular feature of the data is the peak in childlessness for cohorts born between 1901 and 1915. In Appendix A.7 we explain that this peak is related to a change in the Census questionnaire. The Spanish Influenza and the Great Depression may also have played a role (see Sobotka et al. (2011)).

To first address the effect of the increase in education, we keep wages fixed and scale them as to reproduce the average fertility of the generation born in 1941. We then simulate the model setting the shares of men and women in each education category to their historical values for each cohort, keeping everything else constant. The effects on completed fertility of mothers and childlessness are represented by the dashed lines of Figure 8. From the left panel, we observe that the rise in education leads to a monotonous decrease in fertility over time; alone it explains 87\% of the drop in fertility (fertility drops from 4.23 to 2.30 in the data, and from 4.34 to 2.65 in the simulation). As for childlessness, the rise in education predicts a very slight U-shaped pattern for childlessness over time, according to the intuitions developed above. Most of the effect of education is still to increase childlessness due to the increase in the opportunity cost: indeed, in this counterfactual simulation, the wage is fixed at the level reproducing the fertility the 1941 cohort, which is quite high for nineteenth century standards, and this makes very few households affected by social sterility, even at the beginning of the period.

\textsuperscript{25}Doepke et al. (2007) suggest that part of the baby boom is caused by restrictions in the labor market for young women. Albesani and Olivetti (2010) also suggest that improvements in maternal health contributed to the bust. The raise in technological progress in the household sector is also one major cause of the baby boom, as shown in Greenwood et al. (2006).
Secondly, we simulate the effect of a change in the overall wage level, following a trend in TFP (see Appendix A.7.2 for its measurement). To isolate the effect of TFP, we keep education shares constant and equal to the levels of the cohort born in 1941. The dotted lines of Figure 8 show that the rise in TFP implies a monotonic decrease in fertility and a monotonic increase in childlessness.

We next combine the two sources of change in a single simulation. The total effect on fertility, represented by the gray line “edu+tech”, is now too strong, which indicates that other exogenous elements may also have affected fertility. Interestingly, the effect on childlessness becomes strongly U-shaped over time. This is because the effect of the rise in education for the early cohorts is now evaluated at the wage relevant for each period. At the end of the nineteenth century, wages are low and many households are socially sterile. The rise in both education and TFP allow the subsequent generations to escape from that situation and childlessness first drops. Over time, the nature of childlessness changes, becoming more and more the optimal outcome of the fertility decision problem. Ultimately, education increases childlessness. This allows the simulation to capture the slight drop in childlessness over the very long run.

It is particularly interesting to notice that, in the simulation described above, the two driving forces, when taken alone, have either a monotonic increasing effect or a very slight U-shaped effect on childlessness. But once combined, their effect is to bring childlessness down through a non-monotonic path. This highlights the interest of using a structural approach, which has the potential to generate interpretable non-linear effects.

Among the forces that are missing in the above analysis, there is one we can easily incorporate into the model: the fact that part of the non-labor income \( a \) is to be indexed on growth. As \( a \) includes bequests, gifts, welfare transfers, social security etc., it should in the long-run follow TFP developments. The gray line “edu+tech+a” gives the result of a simulation where, in addition to changes in wages and education shares, one third of \( a \) is indexed on TFP. In this simulation, households in the 19th century are less wealthy in terms of non-labor income than in the previous simulation where \( a \) was kept constant at the 1941 level. Hence, their fertility is reduced, and childlessness (social sterility) increased.

In sum, our theory suggests that the U-shaped relationship of childlessness rates over time is caused by a decrease in social sterility followed by an increase in childlessness due to a higher opportunity cost to raise children. Figure 9 confirms this interpretation, showing the breakdown of childlessness for married (left panel) and single (right panel) women separately. Social causes of childlessness have now completely disappeared for married women. This is
Figure 9: Detail of Simulated Childlessness by Cohort, Married (left) and Singles (right) however not true for singles for whom social sterility still exists.\textsuperscript{26}

5.2 Effect of Inequality $\rho$ and Gender Wage Gap, $\gamma$

Historically, there were also changes on the returns to schooling and on the gender wage gap (see Appendix A.7). The effect of these changes on average childlessness and fertility are of second order compared to those of education and TFP. The reason is simply that changes in $\rho$ and $\gamma$ affect choices of the individuals that are in the tails of the distribution of education. As aggregate moments are mainly driven by the middle educated group, the effects of these two parameters are hidden.

Since 1980, income inequality and skill premia have been on the rise in the U.S. To assess the effect on family patterns, we implement a rise in inequality through a change in the Mincer parameter $\rho$ of Equation (5). We increase the return of one additional year of education from 0.092 to 0.126 (see Appendix A.7), keeping the wage of the largest category constant (Category 7); the change in the Mincer parameter thus increases the spread around the wage of high school graduates. This corresponds to an increase in the Gini coefficient computed on $w^f + a^f$ from 0.156 to 0.168.

As the lowly educated become poorer, the minimum consumption constraint binds more often and social sterility rises. For the highly educated, the opportunity cost of having children is increased by the higher skill premium (for both men and women), and more of

\textsuperscript{26}Aaronson et al. (2011) use the variation of the extensive and intensive margins of fertility to assess the effect of a particular education policy in the 1920. In doing so they assume that childlessness is a choice, i.e. it is driven by opportunity costs. This is however questionable, as their study concerns women born around or before 1900, for whom, according to Figure 9, childlessness is mostly related to poverty, and not driven by opportunity cost reasons.
Figure 10: Childlessness Rate (married). Left panel: $\rho = 0.092$ (solid, benchmark); $\rho = 0.126$ (dotted). Right panel: $\gamma = 0.755$ (dashed), $\gamma = 0.869$ (solid), and $\gamma = 1$ (dotted).

them choose to remain childless. We then obtain a more pronounced U-shaped pattern for the higher Mincer coefficient (see left panel of Figure 10). On the whole, the proportion of socially sterile women [S] increases from 2.5% to 3.3%. This suggests that economic changes that make uneducated people better-off might have a positive effect on average fertility because they can affect the intensive margin and the extensive margin of fertility in opposite directions.\footnote{Such opposite effects are discussed in a historical case study by Romaniuk (1980) who shows that, during Zaire’s modernization, fertility increased, partly due to a large reduction in childlessness rates between the 1930’s and the 1960’s.}

Another first order change of the last decades is the closing of the gender wage gap (Goldin (1990) or Jones et al. (2003)). To assess its effect on family patterns, we simulate the model for $\gamma = 0.755$, $\gamma = 0.869$, and $\gamma = 1$, keeping the other parameters fixed. $\gamma = 0.755$ is the lowest value provided in the historical estimates (Appendix A.7). The right panel of Figure 10 shows the effect on childlessness for married women. As expected (Proposition 2 in Appendix B.3), there will be fewer poor women in the social sterility regime, but educated women face a higher opportunity cost of raising children and are more likely to be childless.

6 Conclusion

The main contribution of the paper is to identify several causes behind childlessness that were not accounted for in previous studies of fertility. Childlessness can have natural, social or economic causes. Social sterility appears for women with low education and low non-labor income, either single or married. A woman with an income that is not high enough to allow her to procreate can escape this situation if it is optimal for her husband to abandon part
of his own consumption in order to enable her to produce children within the couple. This
should be highlighted: although aggregate fertility data suggests that Malthusian checks
do not prevail any more and Becker’s model describes the relationship between education
and fertility well, our model detects that some categories of the population are still affected
by Malthusian mechanisms. For highly educated women, childlessness is driven by a high
opportunity cost of childrearing.

Marriage interacts with childlessness in two ways; for poor women, marriage is an opportunity
to get enough resources to be able to have children. Hence, marriage reduces social sterility.
For highly educated women, marriage reduces the opportunity cost of having children, as
husbands also help with raising the children. It therefore also reduces childlessness at the
high end of the female education distribution. The causality also goes from childlessness to
marriage in the sense that because they are more prone to be childless, women at both tails
of education distribution have more difficulties to marry.

Childlessness among educated women does not necessarily call for policy, while social sterility
is a severe limitation of people’s freedom and thus, of particular relevance as far as social
welfare is concerned. Fighting the causes of social sterility would allow the set of capabilities
of the poor to increase. Unfortunately, looking at the data without a structural model does
not allow us to distinguish both types of childlessness, and evaluate their relative importance
in a given society. Our structural approach allows to infer from the behavior of the agents
the unobserved components of childlessness. Hence the importance of combining theory
with data in order to measure the share of childlessness that is related to poverty and/or
inequalities. Our analysis suggests that closing the gender wage gap is a powerful tool for
limiting the proportion of childlessness that is generated by poverty.

References


Paper No. 16146.


Bloom-Feshbach, K., Simonsen, L., Viboud, C., Mølbak, K., Miller, M. A., Gottfredsson, M.
and Andreasen, V. Natality Decline and Miscarriages Associated With the 1918 Influenza


A Data - For Online Publication

A.1 Cohabitation

When dealing with never-married individuals we might be skeptical about whether these individuals are single or just unmarried with a partner (specially poor women who have many children). In the Census 1990, among never-married women aged 45-70, only 1.8% declared themselves as being with a partner. The percentages for mothers and childless were respectively 3.7% and 1.3%. These percentages vary, however, for different education levels: Table 4 provides the proportion of single mothers, aged 45-70, saying they are with an unmarried partner for each education level. This table shows that very few women who have not married are living with a partner. The highest percentages of cohabitation are seen for women aged 36-40, having achieved Grade 1-4, or with a doctoral degree: respectively 21.5% and 21.7% of never-married women in these education levels claim to be unmarried but have a partner.

<table>
<thead>
<tr>
<th>Category</th>
<th>%</th>
<th>Category</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.8</td>
<td>7</td>
<td>3.6</td>
</tr>
<tr>
<td>2</td>
<td>4.8</td>
<td>8</td>
<td>2.8</td>
</tr>
<tr>
<td>3</td>
<td>4.0</td>
<td>9</td>
<td>3.8</td>
</tr>
<tr>
<td>4</td>
<td>3.6</td>
<td>10</td>
<td>3.0</td>
</tr>
<tr>
<td>5</td>
<td>3.8</td>
<td>11</td>
<td>2.8</td>
</tr>
<tr>
<td>6</td>
<td>4.0</td>
<td>12</td>
<td>8.2</td>
</tr>
</tbody>
</table>

Table 4: Percentage of Single Mothers, Aged 45-70, with an Unmarried Partner

According to Akerlof et al. (1996), more than 65% of white women experiencing an out-of-wedlock birth married before the first birthday of the child in 1975. Bumpass and Westoff (1970) estimate that between 1960-64 and 1985-89, around 80% of white women who gave birth to a first child out-of-wedlock married within the ten years that followed. This percentage is lower for black women (a bit larger than 50%). Without focusing on first births, Bramlett and Mosher (2002) estimate that, in 1995, 42.7% of never married women between ages 40-44 had ever cohabited. Their definition for cohabitation was being unmarried but having a sexual relationship while sharing the same usual address. They also say that the probability of transition to marriage after one year of cohabitation is 30%, after 5 years, 70%, and after 10 years, 84%. Consequently, if cohabitation lasts, marriage is very likely to follow.
Rindfuss and VandenHeuvel (1990) compared cohabitants to both married and single
individuals and concluded that cohabitants’ attitudes were in many respects closer to those
of single than of married people. In particular, cohabitants’ fertility expectations are more
similar to those of single than married people. Indeed, comparing the percentage of child-
less individuals who expect children, cohabitants are much closer to never-married than to
married respondents (among women, 11% of cohabiting, 4% of single and 40% of married
people expect to have a child within two years). Consequently, although cohabitation has
many of the characteristics of a marriage (sharing a dwelling unit), cohabitants also share
some of the characteristics of single people (fertility expectations). This puts cohabitants in
a middle position between single and married people.

A.2 Uneducated Single Mothers

Another question raised by the facts concerns the identity of those uneducated mothers who
have many children. Table 5 gives some information. The column \((1 - \phi_n)w^f + a^f\) reports the
total income of the person. Earnings are in column \((1 - \phi_n)w^f\). Not surprisingly, earnings
decrease with the number of children, probably because hours worked drop as the number of
children rises. More interestingly, the column \(a^f\) reports the difference between total income
and earnings. We observe that fertility increases with \(a^f\), which is a prediction of our model.
Now, who are these women? Very few of them report to be with an unmarried partner (see
Table 5). Less than half of them are black. A majority are head of their household.

<table>
<thead>
<tr>
<th>(n)</th>
<th>nobs.</th>
<th>((1 - \phi_n)w^f + a^f)</th>
<th>((1 - \phi_n)w^f)</th>
<th>(a^f)</th>
<th>% with partner</th>
<th>% black</th>
<th>% head of household</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17,531</td>
<td>7,302.87</td>
<td>5,014.92</td>
<td>2,287.95</td>
<td>6.7%</td>
<td>33.5%</td>
<td>35.0%</td>
</tr>
<tr>
<td>2</td>
<td>11,797</td>
<td>7,889.39</td>
<td>5,817.37</td>
<td>2,072.03</td>
<td>9.5%</td>
<td>39.1%</td>
<td>45.3%</td>
</tr>
<tr>
<td>3</td>
<td>7,108</td>
<td>6,923.74</td>
<td>4,553.75</td>
<td>2,369.99</td>
<td>11.1%</td>
<td>39.4%</td>
<td>52.5%</td>
</tr>
<tr>
<td>4</td>
<td>4,636</td>
<td>6,712.25</td>
<td>3,998.48</td>
<td>2,713.77</td>
<td>6.4%</td>
<td>32.1%</td>
<td>56.8%</td>
</tr>
<tr>
<td>5</td>
<td>2,882</td>
<td>6,239.59</td>
<td>3,556.02</td>
<td>2,683.57</td>
<td>5.0%</td>
<td>41.8%</td>
<td>61.1%</td>
</tr>
<tr>
<td>6+</td>
<td>5,446</td>
<td>5,844.52</td>
<td>2,576.98</td>
<td>3,267.54</td>
<td>6.1%</td>
<td>36.4%</td>
<td>64.5%</td>
</tr>
</tbody>
</table>

Table 5: Single Mothers with no Education

A.3 Five-year cohorts

The stylized facts highlighted in Section 2 can also be found if we consider each 5-years cohort
separately. Figures 11 and 12 show the relationship between childlessness and education and
the fertility of mothers and education for married and single women for each cohort. The
only major difference between cohorts is in the childlessness rate of single women, with intermediate levels of education (Figure 12): older single women were much less likely to be mothers than younger single women.

Figure 11: Childlessness Rate and Completed Fertility of Mothers, by Education Category and by Age, Married Women

Figure 12: Childlessness Rate and Completed Fertility of Mothers, by Education Category and by Age, Single Women

A.4 Standard errors of the mean

Table 6 reports the standard errors of the mean for the completed fertility and the childlessness rates of both single and married mothers.
<table>
<thead>
<tr>
<th>Education Category</th>
<th>Childlessness Rates</th>
<th>Completed Fertility of Mothers</th>
<th>Marriage Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Married Single</td>
<td>Married Single</td>
<td>Women Men</td>
</tr>
<tr>
<td>1</td>
<td>0.70 1.61</td>
<td>7.49 22.05</td>
<td>0.941 0.849</td>
</tr>
<tr>
<td>2</td>
<td>0.55 2.35</td>
<td>6.30 21.54</td>
<td>0.717 0.524</td>
</tr>
<tr>
<td>3</td>
<td>0.23 1.29</td>
<td>2.06 11.58</td>
<td>0.231 0.194</td>
</tr>
<tr>
<td>4</td>
<td>0.32 2.27</td>
<td>2.62 16.76</td>
<td>0.297 0.286</td>
</tr>
<tr>
<td>5</td>
<td>0.25 2.03</td>
<td>1.98 15.69</td>
<td>0.221 0.246</td>
</tr>
<tr>
<td>6</td>
<td>0.25 2.29</td>
<td>2.09 16.26</td>
<td>0.231 0.271</td>
</tr>
<tr>
<td>7</td>
<td>0.09 0.60</td>
<td>0.57 06.21</td>
<td>0.074 0.089</td>
</tr>
<tr>
<td>8</td>
<td>0.15 0.83</td>
<td>0.86 09.38</td>
<td>0.126 0.114</td>
</tr>
<tr>
<td>9</td>
<td>0.27 1.60</td>
<td>1.55 16.15</td>
<td>0.225 0.226</td>
</tr>
<tr>
<td>10</td>
<td>0.23 0.62</td>
<td>1.07 15.56</td>
<td>0.199 0.144</td>
</tr>
<tr>
<td>11</td>
<td>0.35 0.47</td>
<td>1.41 22.01</td>
<td>0.346 0.175</td>
</tr>
<tr>
<td>12</td>
<td>1.47 1.34</td>
<td>5.08 48.75</td>
<td>1.398 0.403</td>
</tr>
</tbody>
</table>

Table 6: Standard Errors ($\times 10^{-3}$) from U.S. Census 1990

A.5 Race, Ethnicity and Place of birth

A.5.1 Race and Ethnicity

In this first part of the appendix, we split the population into five groups, Whites, Blacks, Natives, Asians and Hispanics. We constructed each group from the two variables RACE and HISPAN. In order for an individual to be considered as White, Black, Native or Asian, he or she has to be from that particular race and “not Hispanic”. Table 7 gives the number of observations (unweighted) by education category for each group. Figures 13 and 14 show the childlessness rate and completed fertility of mothers, by education category, for married women and single women respectively. Figure 15 shows marriage rates for men and women for each group, by education category.

For all groups, we find a negative relationship between the fertility of married mothers and education. For single mothers, we have few observations for Asians and Natives so the relationship is not as clear. For other single mothers, the relationship holds for Blacks and Hispanics and is slightly hump-shaped for Whites. The fertility differential between single and married mothers is larger for white women than for others: married mothers have
<table>
<thead>
<tr>
<th></th>
<th>Blacks</th>
<th>Whites</th>
<th>Natives</th>
<th>Asians</th>
<th>Hispanics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,264</td>
<td>4,631</td>
<td>274</td>
<td>1,518</td>
<td>4,400</td>
</tr>
<tr>
<td>2</td>
<td>1,673</td>
<td>5,276</td>
<td>200</td>
<td>868</td>
<td>6,002</td>
</tr>
<tr>
<td>3</td>
<td>8,897</td>
<td>59,603</td>
<td>806</td>
<td>2,397</td>
<td>12,487</td>
</tr>
<tr>
<td>4</td>
<td>4,029</td>
<td>29,879</td>
<td>300</td>
<td>832</td>
<td>3,059</td>
</tr>
<tr>
<td>5</td>
<td>5,468</td>
<td>47,968</td>
<td>471</td>
<td>680</td>
<td>2,598</td>
</tr>
<tr>
<td>6</td>
<td>5,877</td>
<td>40,798</td>
<td>361</td>
<td>338</td>
<td>2,018</td>
</tr>
<tr>
<td>7</td>
<td>24,421</td>
<td>429,963</td>
<td>1,985</td>
<td>8,115</td>
<td>15,078</td>
</tr>
<tr>
<td>8</td>
<td>8,787</td>
<td>160,406</td>
<td>937</td>
<td>2,988</td>
<td>5,104</td>
</tr>
<tr>
<td>9</td>
<td>2,631</td>
<td>46,909</td>
<td>275</td>
<td>1,758</td>
<td>1,838</td>
</tr>
<tr>
<td>10</td>
<td>4,092</td>
<td>88,089</td>
<td>262</td>
<td>4,551</td>
<td>2,025</td>
</tr>
<tr>
<td>11</td>
<td>3,641</td>
<td>49,819</td>
<td>164</td>
<td>1,941</td>
<td>1,283</td>
</tr>
<tr>
<td>12</td>
<td>242</td>
<td>3,974</td>
<td>16</td>
<td>207</td>
<td>172</td>
</tr>
<tr>
<td>Total</td>
<td>71,022</td>
<td>967,315</td>
<td>6,051</td>
<td>26,193</td>
<td>56,064</td>
</tr>
</tbody>
</table>

Note: We do not know the race or ethnicity of 435 observations so the sum of the five groups is not equal to the sum of the observations of Table 2.

Table 7: Number of Observations by Education Category and by Race or Ethnicity

Figure 13: Childlessness Rate and Completed Fertility of Mothers, by Education Category, Married Women
Figure 14: Childlessness Rate and Completed Fertility of Mothers, by Education Category, Single Women

Figure 15: Marriage Rates for Female and Male, by Education Category, Single Women
between 1.5 and 2 times more children than single mothers. Both the U-shaped relationship between childlessness and education and the hump-shaped relationship between marriage rates and education hold in general. In particular for Asian married women, childlessness always increases with education; childlessness of single Hispanic women is flat for the first education levels and then increases. Comparing across groups, we see that Black women are in general more likely to be childless if married, less likely to be childless if single and that the U-shaped relationship (Fact 2) is more pronounced for them than for other groups. Both single and married Black mothers have more children than White mothers. Differential fertility between groups decreases with education.

A.5.2 Place of birth

Some education groups in our sample could comprise large shares of immigrants who may have made their marriage and fertility decisions in another country. It could then be the case that some of our facts do not hold when withdrawing these persons. To assess this, we divide the population into two groups: those who were born in the U.S. and those who were born abroad. The place of birth is obtained thanks to the variable “Birthplace” (BPL) in IPUMS. The percentage of persons born in the U.S. in our sample is 90.96% (so only 9.04% were born abroad). Interestingly, as Table 8 indicates, the main group is not Hispanic (Central and Southern America) but European.

<table>
<thead>
<tr>
<th>Continent</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa</td>
<td>0.009</td>
</tr>
<tr>
<td>Asia</td>
<td>0.220</td>
</tr>
<tr>
<td>Central America and Caribbean</td>
<td>0.265</td>
</tr>
<tr>
<td>Europe</td>
<td>0.356</td>
</tr>
<tr>
<td>Southern America</td>
<td>0.047</td>
</tr>
<tr>
<td>Oceania</td>
<td>0.038</td>
</tr>
<tr>
<td>Other north America</td>
<td>0.066</td>
</tr>
</tbody>
</table>

Table 8: Distribution of foreign born persons by continent of birth

Removing foreign born individuals from the sample results in the moments shown in Table 9. The facts highlighted in Section 2 are robust to the deletion of these observations. In particular, we see that the relationship between childlessness rates and education for single women is even more U-shaped than in the benchmark case which includes foreign born persons.
Table 9: Facts from U.S. Census 1990 without foreign born persons

Table 10 shows the facts for the individuals that were born abroad only. We can observe that the relationship between marriage rates of males and education is no longer hump-shaped but rather high and flat. Fertility of mothers still decreases with education. We also find that childlessness rates are monotonously increasing with education. We suspect that, this absence of U-shaped relationship between childlessness and education among foreign born persons, is due to a selection effect. Indeed, the persons who are included in the Census are those who succeeded in emigrating to the U.S.. They are probably in better health and have higher non-labor income than the persons without education born in the U.S.. Then, they are less concerned with social sterility.

A.6 Disability

The U-shaped relationship between childlessness and education is highly affected when we consider the data without individuals who have any lasting physical or mental health condition that prevents or causes difficulty working, living independently or taking care of their own personal needs (respectively, variables DISABWRK, DIFFMOB and DIFFCARE of IPUMS). Note that these variables say nothing about the ability to reproduce.\textsuperscript{28} For mar-

\textsuperscript{28}See \url{http://www.census.gov/hhes/www/disability/sipp/disab02/ds02t2.html} for information about disability types and their proportions in the U.S.
Table 10: Facts from U.S. Census 1990 including only foreign born persons - Total number of observations (unweighted) 97135

For example, a person affected by the Down Syndrome, which is one of the most common disabilities, life expectancy was below 18 in the thirties (while it is nowadays around 50 years). In the second group, we have those who became disabled because of poverty. Either because they worked or lived in an unhealthy or dangerous environment or because they had a bad medical treatment, or none, and remained single and childless. These two factors are likely to affect their health aged 45-70. This leads to an endogeneity problem in the relationship between disability and education: adults who lived in the worst conditions (the ones with the lowest education) are the most likely to suffer health problems when older.
In other words, the lowly educated are more likely to have health problems when old than the highly educated. In this group, disabled women are, *de facto*, lowly educated and the constraint \( c^f \geq \hat{c} \) reflects their incapacity to have children when they do not have a husband investing in them. Our position is that we should keep this group in the sample.

### A.7 Historical Data

#### A.7.1 IPUMS data

In the historical exercise we also use Census data, taken from IPUMS. Table 11 lists the 22 cohorts that were considered as well as the Census from which they were constructed. As the fertility of single women was not asked before 1970, the data is only for ever-married women.

Two remarks before analyzing completed fertility over time should be noticed. First, there was a change on how the number of children ever born was reported. First, for the 1940 Census, there was only one case for the number of children that a woman had had while in later Censuses a case for “none” was added to the questionnaire - see Figure 16. This could then lead many childless women in cohorts 1971-1900 to be counted as not respondents instead of as childless and hence explain partly the increase in childlessness rates for the following cohorts. This was already pointed out in Grabill and Glick (1959).

A second aspect concerns the high childlessness rates for the cohorts of women born at the beginning of last century. These women have been touched by the 1918 Spanish Influenza.

Figure 16: Question on Children Ever Born in 1940, 50, 60, 70, 80 and 90 censuses
<table>
<thead>
<tr>
<th>Cohort</th>
<th>Age</th>
<th>Census</th>
<th>Completed Fertility of Mothers</th>
<th>Childlessness Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1871-75</td>
<td>65-69</td>
<td>1940</td>
<td>4.228</td>
<td>0.164</td>
</tr>
<tr>
<td>1876-80</td>
<td>60-64</td>
<td>1940</td>
<td>3.998</td>
<td>0.171</td>
</tr>
<tr>
<td>1881-85</td>
<td>55-59</td>
<td>1940</td>
<td>3.915</td>
<td>0.169</td>
</tr>
<tr>
<td>1886-90</td>
<td>50-54</td>
<td>1940</td>
<td>3.758</td>
<td>0.158</td>
</tr>
<tr>
<td>1891-95</td>
<td>45-49</td>
<td>1940</td>
<td>3.585</td>
<td>0.150</td>
</tr>
<tr>
<td>1896-00</td>
<td>40-44</td>
<td>1940</td>
<td>3.353</td>
<td>0.156</td>
</tr>
<tr>
<td>1901-05</td>
<td>45-49</td>
<td>1950</td>
<td>3.202</td>
<td>0.189</td>
</tr>
<tr>
<td>1906-10</td>
<td>40-44</td>
<td>1950</td>
<td>2.885</td>
<td>0.203</td>
</tr>
<tr>
<td>1911-15</td>
<td>45-49</td>
<td>1960</td>
<td>2.917</td>
<td>0.176</td>
</tr>
<tr>
<td>1916-20</td>
<td>40-44</td>
<td>1960</td>
<td>2.987</td>
<td>0.132</td>
</tr>
<tr>
<td>1921-25</td>
<td>45-49</td>
<td>1970</td>
<td>3.152</td>
<td>0.100</td>
</tr>
<tr>
<td>1926-30</td>
<td>40-44</td>
<td>1970</td>
<td>3.372</td>
<td>0.081</td>
</tr>
<tr>
<td>1931-35</td>
<td>45-49</td>
<td>1980</td>
<td>3.448</td>
<td>0.070</td>
</tr>
<tr>
<td>1936-40</td>
<td>40-44</td>
<td>1980</td>
<td>3.237</td>
<td>0.066</td>
</tr>
<tr>
<td>1941-45</td>
<td>45-49</td>
<td>1990</td>
<td>2.786</td>
<td>0.082</td>
</tr>
<tr>
<td>1946-50</td>
<td>40-44</td>
<td>1990</td>
<td>2.471</td>
<td>0.102</td>
</tr>
<tr>
<td>1954-58</td>
<td>40-44</td>
<td>1998</td>
<td>2.320</td>
<td>0.137</td>
</tr>
<tr>
<td>1956-60</td>
<td>40-44</td>
<td>2000</td>
<td>2.367</td>
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<td>2002</td>
<td>2.359</td>
<td>0.120</td>
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<td>1960-64</td>
<td>40-44</td>
<td>2004</td>
<td>2.357</td>
<td>0.132</td>
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<td>1962-66</td>
<td>40-44</td>
<td>2006</td>
<td>2.345</td>
<td>0.146</td>
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<td>40-44</td>
<td>2008</td>
<td>2.297</td>
<td>0.120</td>
</tr>
</tbody>
</table>

Note: Data for cohorts born after 1954 are taken from Tables available at the Census webpage.

Table 11: Cohorts for the Historical Exercise and Completed Fertility of Mothers and Childlessness Rates for Married Women of Each Cohort

Pregnant women were particularly vulnerable to the Spanish Influenza. If they were affected by the flu, they had high chances to either die or have a miscarriage (Bloom-Feshbach et al. (2011)).

Morgan (1991) also shows childlessness data using vital registration estimates. His estimates for the cohorts that were born at the beginning of the 1900s are lower than the average rates from the Census (see Figure 1 in Morgan (1991)). Our estimates of childlessness rates are close to those of Figure A2 in Jones and Tertilt (2008).
A.7.2 TFP

Data on TFP are taken from Chari et al. (2006) for the period 1901-2000 and from Conference Board (2013) for the period 2000-2010. We compute 30-years averages to represent the mean life-cycle wage (for example, the wage of the cohort born in 1871 is indexed on average TFP between 1901 and 1931). For the cohorts born in the 50s and 60s, we forecast future TFP assuming the same growth rate as the one observed in the twentieth century (average annual growth rate of 0.47%).

A.7.3 Changes in Education

To compute changes in education, we divided the sample of each cohort into the 12 categories of education. For cohorts generated from Censuses before 1900, we cannot distinguish between categories 11 and 12 so they were merged. The codes (from the variable EDUCD) that were used for each Census are provided in Table 12. Results are provided in Tables 13 and 14.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
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<td>2</td>
<td>2</td>
<td>2</td>
</tr>
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<td>14-17</td>
<td>13</td>
<td>10</td>
<td>14-17</td>
</tr>
<tr>
<td>3 Grade 5-8</td>
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<td>20</td>
<td>21, 24</td>
<td>22, 23, 25, 26</td>
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<td>30</td>
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<td>5 Grade 10</td>
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<td>40</td>
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<td>7 Grade 12</td>
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<td>61, 62</td>
<td>61, 63, 64</td>
</tr>
<tr>
<td>8 1 year of college</td>
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<td>71</td>
<td>65, 71</td>
<td>65, 71</td>
</tr>
<tr>
<td>9 2 years of college</td>
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<td>81</td>
<td>81</td>
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<td>114, 115</td>
<td>114, 115</td>
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Note: Data for years 2010-2008 is taken from the American Community Survey (ACS). As Census 1998 is not available in IPUMS, we used Census 2000 for the education shares of cohort 1958-1958.

Table 12: Codes for Education Shares for Each Cohort (from EDUCD variable)
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<th>Cohort</th>
<th>Age</th>
<th>Census</th>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<th>9</th>
<th>10</th>
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<th>12</th>
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<tbody>
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<td>65-69</td>
<td>1940</td>
<td>0.062</td>
<td>0.120</td>
<td>0.559</td>
<td>0.036</td>
<td>0.038</td>
<td>0.017</td>
<td>0.097</td>
<td>0.013</td>
<td>0.022</td>
<td>0.030</td>
<td>0.005</td>
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<tr>
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<td>60-64</td>
<td>1940</td>
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<td>0.539</td>
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<td>0.044</td>
<td>0.022</td>
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<td>0.019</td>
<td>0.032</td>
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<td>0.053</td>
<td>0.049</td>
<td>0.021</td>
<td>0.116</td>
<td>0.014</td>
<td>0.021</td>
<td>0.032</td>
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<td>0.051</td>
<td>0.024</td>
<td>0.123</td>
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<td>1940</td>
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<td>0.490</td>
<td>0.057</td>
<td>0.061</td>
<td>0.027</td>
<td>0.134</td>
<td>0.019</td>
<td>0.029</td>
<td>0.039</td>
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Note: Education categories 11 and 12 were merged before 1970.

Table 13: Proportion of Women in Each Education Category
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<th>Census</th>
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<td>0.012</td>
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<td>0.016</td>
<td>0.033</td>
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<td>0.010</td>
<td>0.050</td>
<td>0.026</td>
<td>0.034</td>
<td>0.028</td>
<td>0.301</td>
<td>0.192</td>
<td>0.058</td>
<td>0.150</td>
<td>0.117</td>
<td>0.024</td>
</tr>
<tr>
<td>1946-1946</td>
<td>40-44</td>
<td>1990</td>
<td>0.010</td>
<td>0.008</td>
<td>0.035</td>
<td>0.035</td>
<td>0.018</td>
<td>0.024</td>
<td>0.021</td>
<td>0.260</td>
<td>0.214</td>
<td>0.078</td>
<td>0.191</td>
<td>0.125</td>
</tr>
<tr>
<td>1954-1958</td>
<td>42-46</td>
<td>2000</td>
<td>0.012</td>
<td>0.005</td>
<td>0.030</td>
<td>0.016</td>
<td>0.021</td>
<td>0.022</td>
<td>0.306</td>
<td>0.219</td>
<td>0.072</td>
<td>0.179</td>
<td>0.102</td>
<td>0.015</td>
</tr>
<tr>
<td>1956-1960</td>
<td>40-44</td>
<td>2000</td>
<td>0.012</td>
<td>0.005</td>
<td>0.030</td>
<td>0.017</td>
<td>0.022</td>
<td>0.024</td>
<td>0.318</td>
<td>0.216</td>
<td>0.071</td>
<td>0.177</td>
<td>0.095</td>
<td>0.013</td>
</tr>
<tr>
<td>1958-1962</td>
<td>40-44</td>
<td>2002</td>
<td>0.007</td>
<td>0.008</td>
<td>0.030</td>
<td>0.016</td>
<td>0.022</td>
<td>0.025</td>
<td>0.332</td>
<td>0.062</td>
<td>0.204</td>
<td>0.189</td>
<td>0.092</td>
<td>0.014</td>
</tr>
<tr>
<td>1960-1964</td>
<td>40-44</td>
<td>2004</td>
<td>0.007</td>
<td>0.006</td>
<td>0.033</td>
<td>0.016</td>
<td>0.023</td>
<td>0.027</td>
<td>0.337</td>
<td>0.112</td>
<td>0.082</td>
<td>0.209</td>
<td>0.101</td>
<td>0.017</td>
</tr>
<tr>
<td>1962-1966</td>
<td>40-44</td>
<td>2006</td>
<td>0.009</td>
<td>0.008</td>
<td>0.036</td>
<td>0.019</td>
<td>0.023</td>
<td>0.025</td>
<td>0.344</td>
<td>0.134</td>
<td>0.081</td>
<td>0.202</td>
<td>0.102</td>
<td>0.016</td>
</tr>
<tr>
<td>1964-1968</td>
<td>40-44</td>
<td>2008</td>
<td>0.012</td>
<td>0.008</td>
<td>0.036</td>
<td>0.019</td>
<td>0.022</td>
<td>0.027</td>
<td>0.286</td>
<td>0.159</td>
<td>0.082</td>
<td>0.220</td>
<td>0.110</td>
<td>0.017</td>
</tr>
</tbody>
</table>

Note: Education categories 11 and 12 were merged before 1970.

Table 14: Proportion of Men in Each Education Category
A.7.4 Changes in Wages

For each cohort we run the following linear regression for single individuals with a positive income wage (INCWAGE variable in the Census):

\[
\ln(\text{INCWAGE}) = \text{constant} + \ln(\gamma) i + \rho e
\]  

(6)

where \(i = 0\) if the individual is a man and \(i = 1\) if she is a woman. \(e\) are the years of schooling that we give to each category of education (Table 2). The estimated parameters \(\ln(\gamma)\) and \(\rho\) denote the natural logarithm of the gender wage gap and the mincer coefficient respectively. Table 15 provides the estimated values for \(\gamma\) and \(\rho\) with the respective standard errors (SE) for each cohort. The values of the estimates that we use in Section 4.1 are those of the cohort 1941-1945 (in bold).

<table>
<thead>
<tr>
<th>Cohort</th>
<th>(\gamma)</th>
<th>SE</th>
<th>(\rho)</th>
<th>SE</th>
<th>Cohort</th>
<th>(\gamma)</th>
<th>SE</th>
<th>(\rho)</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1871-1875</td>
<td>0.988</td>
<td>(0.0076)</td>
<td>0.092</td>
<td>(0.0009)</td>
<td>1926-1930</td>
<td>0.755</td>
<td>(0.0020)</td>
<td>0.073</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>1876-1880</td>
<td>0.928</td>
<td>(0.0053)</td>
<td>0.093</td>
<td>(0.0007)</td>
<td>1931-1935</td>
<td>0.816</td>
<td>(0.0025)</td>
<td>0.080</td>
<td>(0.0004)</td>
</tr>
<tr>
<td>1881-1885</td>
<td>0.927</td>
<td>(0.0041)</td>
<td>0.085</td>
<td>(0.0005)</td>
<td>1936-1940</td>
<td>0.787</td>
<td>(0.0022)</td>
<td>0.086</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>1886-1890</td>
<td>0.891</td>
<td>(0.0033)</td>
<td>0.083</td>
<td>(0.0004)</td>
<td>1941-1945</td>
<td>0.869</td>
<td>(0.0019)</td>
<td>0.092</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>1891-1895</td>
<td>0.859</td>
<td>(0.0028)</td>
<td>0.072</td>
<td>(0.0004)</td>
<td>1946-1950</td>
<td>0.904</td>
<td>(0.0013)</td>
<td>0.094</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>1896-1900</td>
<td>0.805</td>
<td>(0.0023)</td>
<td>0.090</td>
<td>(0.0003)</td>
<td>1954-1958</td>
<td>0.893</td>
<td>(0.0011)</td>
<td>0.111</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>1901-1905</td>
<td>0.787</td>
<td>(0.0025)</td>
<td>0.057</td>
<td>(0.0003)</td>
<td>1956-1960</td>
<td>0.877</td>
<td>(0.0015)</td>
<td>0.112</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>1906-1910</td>
<td>0.843</td>
<td>(0.0021)</td>
<td>0.046</td>
<td>(0.0003)</td>
<td>1958-1962</td>
<td>0.843</td>
<td>(0.0010)</td>
<td>0.119</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>1911-1915</td>
<td>0.767</td>
<td>(0.0023)</td>
<td>0.076</td>
<td>(0.0003)</td>
<td>1960-1964</td>
<td>0.841</td>
<td>(0.0011)</td>
<td>0.126</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>1916-1920</td>
<td>0.772</td>
<td>(0.0019)</td>
<td>0.073</td>
<td>(0.0003)</td>
<td>1962-1966</td>
<td>0.848</td>
<td>(0.0009)</td>
<td>0.125</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>1921-1925</td>
<td>0.789</td>
<td>(0.0022)</td>
<td>0.066</td>
<td>(0.0003)</td>
<td>1964-1968</td>
<td>0.836</td>
<td>(0.0009)</td>
<td>0.112</td>
<td>(0.0002)</td>
</tr>
</tbody>
</table>

Note: As Census 1998 is not available in IPUMS, we used Census 2000 to construct cohort 1954-1958.

Table 15: Estimates for the Gender Wage Gap and Returns to Schooling for Cohorts born between 1971-1968

A.8 Returns to schooling

Individuals in the extreme education levels are relatively few. This could imply that Mincer equations linking economic returns to schooling works well for the middle of the distribution of wages, but is less adequate for the tails of the distribution. As the tails are important to reproduce the U-shaped childlessness pattern, we looked at this question in detail.

We estimate a Mincer equation by regressing the natural logarithm of the variable “wage and salary income (INCWAGE)” on a constant plus the years of education for our sample
of women who are still working (UHRSWORK > 0). Results are provided in Table 16. We see that the regression coefficients are significant at the 1% level and close to 0.1.

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Married All</th>
<th>Married Childless</th>
<th>Single All</th>
<th>Single Childless</th>
<th>All</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Years of Education</td>
<td>0.097</td>
<td>0.113</td>
<td>0.111</td>
<td>0.109</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td>(0.000137)</td>
<td>(0.000451)</td>
<td>(0.000310)</td>
<td>(0.000365)</td>
<td>(0.000126)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>8.019</td>
<td>7.961</td>
<td>8.157</td>
<td>8.210</td>
<td>8.005</td>
<td>8.005</td>
</tr>
</tbody>
</table>

Note: The dependent variable is the logarithm of the variable “wage and salary income”. We only show results for women in our sample for whom the variable “usual hours worked per week” was positive.

Table 16: Univariate Estimates of the Returns to Schooling

Figure 17 plots the estimates of the variable “wage and salary income” together with the mean observations for different years of education, for married and single women respectively (childless and mothers). In both cases, the fit for highly educated women is good, so we do not expect biases for the right tail of the wage distribution. For low skilled women results are mixed. For married women, the exponential function clearly underestimate the earnings of the low education women. For singles, however, the fit remains good.

As most women who are concerned by social sterility are single women, our estimate of this type of childlessness is robust to alternative specifications of the wage function that would track the observed wages even better. For married women, on the contrary, our specification is too pessimistic concerning their income, and we may overestimate the proportions of the women in the regimes of social sterility [S] and constrained fertility [C].
As fertility decisions affect the number of hours worked and consequently, earnings, we remove this endogeneity by reestimating the Mincer equation on a subsample, including only the childless women. Results are presented in Figure 18. We find the same result as above, that the exponential relationship between years of schooling and wages underestimates the wages for low educated married women, but does fine for single women.

Figure 18: Earnings as a Function of Education among Childless. Married (left), Single (right)
B Theory

In this Appendix, we solve the third stages of the game, which is the maximization problem of a single women and of a couple. Before studying the precise conditions on wages and non-labor income under which the various constraints bind, we list the different possible “regimes”, describing different living arrangements.

B.1 Possible regimes

Table 17 summarizes consumption and fertility choices in each regime, where we define for married couples:

\[ a = a^f + a^m - \mu. \]

B.1.1 Single Women

A single woman maximizes (1) subject to (2), (3), \( b^f(c^f, n) \leq 0 \) and \( 0 \leq n \leq n_M \). She can be in different regimes, depending on which constraint is binding.

Regime 0. [N] If a single woman is naturally sterile, she consumes her total income net of costs. Her consumption can be either smaller or greater than \( \hat{c} \) as natural sterility is not correlated to education and earnings.

Regime I. [U] If no other constraint than the budget constraint is binding and the corner solution with no children does not dominate, the unconstrained fertility regime prevails. The equation of \( n_I \) shows that the effect of the woman’s education on fertility is ambiguous. A higher \( w_f \) implies the usual income effect which raises fertility. But the increase in \( w_f \) also raises the time cost to rearing children, which reduces fertility through a substitution effect. If \( a_f > \mu \), the substitution effect dominates and \( n_I \) decreases with \( w_f \), while the reverse is true if \( a_f < \mu \).

Regime II. [S] If her income does not allow the consumption level required to have children to be reached, the single woman lives in poverty and is prevented from choosing to have children.
Single Men

\[ c^m = (1 - \delta^m)w^m + a^m - \mu \]

Single Women

\[ c_0^f = (1 - \delta^f)w^f + a^f - \mu \lesssim \hat{c} \]
\[ c_1^f = \frac{w^f(1 - \delta^f + \phi(\nu - \eta)) + a^f - \mu}{2} \]
\[ c_{II}^f = (1 - \delta^f)w^f + a^f - \mu < \hat{c} \]
\[ c_{III}^f = \hat{c} \]
\[ c_{IV}^f = (1 - \delta^f)w^f + a^f - \mu \]

Couples

\[ c_{V}^f = \frac{\theta y}{2} > \frac{\hat{c}}{2} \]
\[ c_{VI}^f = \hat{\theta}(w^m + w^f + a) < \frac{\hat{c}}{2} \]
\[ c_{VIII}^f = \hat{\theta} \left( w^m + w^f + a \right) \]
\[ c_{IX}^f = \hat{\theta}(w^m + w^f + a) \]
\[ c_{X}^f = \hat{\theta} \left( \frac{2\alpha - 1}{\alpha} \right) w^m + a - \frac{\hat{c}}{2} \]
\[ c_{XI}^f = \hat{\theta} \left( \frac{2\alpha - 1}{\alpha} \right) w^m + a \]
\[ n_0 = 0 \]
\[ n_1 = \frac{1}{2} \left[ \frac{w^f(1 - \delta^f + \phi(\nu - \eta)) + a^f - \mu}{\hat{c}^f} \right] - \nu. \]
\[ n_{II} = 0 \]
\[ n_{III} = \frac{w^f(1 - \delta^f - \phi\eta) + a^f - \hat{c} - \mu}{\hat{c}^f} \]
\[ n_{IV} = 0 \]
\[ n_{V} = 2M \]

\[ n_{VI} = \frac{y}{2\phi(\alpha w^f + (1 - \alpha)w^m)} - \nu > 0 \]
\[ n_{VII} = 0 \]
\[ n_{VIII} = \frac{1}{2 - \hat{\theta}} \left[ \frac{y - \hat{c}}{\phi(\alpha w^f + (1 - \alpha)w^m)} \right] - \nu \]
\[ n_{IX} = 0 \]
\[ n_{X} = \tilde{n}_M \]
\[ n_{XI} = \tilde{n}_M \]
\[ n_{XII} = 0 \]

Note: \( y = w^f (1 + \phi\alpha(\nu - \eta)) + w^m (1 + \phi(1 - \alpha)(\nu - \eta)) + a, \) \( \hat{\theta} = \theta(w^f, w^m) \)

Table 17: Possible Regimes
Regime III. [C] If her income is sufficiently high to escape Regime II but not high enough to have as many children as in Regime I, it may be optimal for the single woman to work and consume more in order to be able to procreate. A necessary condition for this regime to prevail is \( a^f < \hat{c} + \mu \). Otherwise the woman would be able to consume more than \( \hat{c} \) without working. \( a^f < \hat{c} + \mu \) implies that \( n_{III} \) increases with the wage \( w^f \). Regime III echoes Malthus’s fertility theory (see e.g. Galor (2005)).

Regime IV. [O] When being childless yields the highest utility and the minimal consumption constraint (2) does not bind, childlessness prevails. This regime arises when the opportunity cost of having children is high and the single woman fully specializes in labor market activities.

Regime V. [M] When the opportunity cost of having children is low and the non-labor income is high, it may be optimal for the woman to fully specialize in the production of children.

B.1.2 Couples

The problem for a couple is

\[
\max_{c^f,c^m,n} \theta(w^f,w^m) \ln c^f + (1 - \theta(w^f,w^m)) \ln c^m + \ln(\nu + n)
\]

subject to (2), (3), (3) and \( 0 \leq n \leq \bar{n}_M \).\(^{29}\) Six regimes are possible when none of the spouses are naturally sterile. When one of them is naturally sterile, the couple lives Regime XII described hereafter.

Regime VI. [U] If no constraint is binding and the corner solution with no children does not dominate we have the unconstrained fertility regime. An increase in the wage of the husband has an ambiguous effect on fertility:

\[
\frac{\partial n_{VI}}{\partial w^m} > 0 \iff w^f > \frac{1 - \alpha}{2\alpha - 1} a.
\]  

\(^{29}\) An alternative could be to add two participation constraints: \( u^f(married) \geq u^f(single) \) and \( u^m(married) \geq u^m(single) \). This implies that the partner with the highest negotiation power can have an interest in reducing his/her welfare in order to incite his/her match to accept marriage. However, because rationality is common knowledge, such a marriage contract would not be credible: as couples cannot divorce, the partner with the highest negotiation power has an incentive to deviate from the agreement.
In most of the literature (Galor and Weil (1996), etc.) \( \alpha = 1 \), then an increase in the wage of the husband has a pure income effect and \( \frac{\partial n_{v1}}{\partial w^m} > 0 \). When \( \alpha < 1 \), the income effect always dominates the substitution effect in families with very low non-labor incomes \( (a < 0) \). In families with higher non-labor incomes \( (a > 0) \), the substitution effect dominates only when the wage of the woman is low relative to the non-labor income of the couple.

In the case of an increase in the wife’s wage, we find:

\[
\frac{\partial n_{v1}}{\partial w^f} < 0 \iff w^m > \frac{\alpha}{1 - 2\alpha} a.
\]

As long as \( \alpha > 1/2 \), an increase in the woman’s wage always reduces fertility when \( a > 0 \). In families with very low non-labor income \( (a < 0) \), the income effect dominates if the wage of the husband is also very low.\(^{30}\)

Parameter \( \alpha \) is related to the slope of the wage-fertility relationship. Higher \( \alpha \) implies a more negative link between mother’s education and fertility, together with a less negative link between father’s education and fertility. We will come back to this result in the section on identification (Section 4.2). Finally, \( n_{v1} \) does not depend on the negotiation power \( \theta(w^f, w^m) \) as there are no gender differences in preferences for children.

**Regime VII. [S]** When the total net income is too low to guarantee a sufficient consumption level to procreate, even if the husband would not consume, the couple is childless due to poverty.

**Regime VIII. [C]** When the bargaining power of the wife is too low to guarantee a sufficient consumption level allowing her to procreate in the interior regime, it can be optimal for the husband to give up some consumption in order to have children. This is the only regime in which fertility depends on the wife’s bargaining power. This makes fertility subject to three effects: the usual income and substitution effects and a negotiation power effect. The latter arises because an increase in the wife’s education gives her more bargaining power. As, in this regime, her consumption is fixed, the only way to obtain more utility is through increased fertility. The net effect on fertility is the result of these three forces. Notice that once \( w^f \) becomes sufficiently high, the couple will be able to have children in Regime VI. This is due to both the increased bargaining power of the wife allowing her to benefit more from the marriage surplus and to an income effect: as \( w^f \) increases, the total income of the

\(^{30}\)Jones and Schoonbroodt (2010) provide a discussion on the possible impacts of women’s wages on fertility.
couple increases allowing both spouses to consume more. This regime does not exist for a single woman as it relies on the additional income provided by the husband.

**Regime IX. [O]** When choosing to be childless yields the highest utility, the couple chooses to have no children. In this case, both spouses specialize in labor market activities.

**Regime X. [C] and [M]** When it is optimal for the husband to give up some of his consumption for his wife to specialize entirely in procreation, the couple has the maximum number of children. Compared to Regime VIII, the optimal fertility rate of the couple does not depend on the negotiation power of the wife because she is not able to give birth to an additional child to increase her utility.

**Regime XI. [M]** When the couple is sufficiently rich (either from non-labor income or from husband education) for the wife to specialize in childrearing without requiring any sacrifice from the husband, the couple has the maximum fertility.

**Regime XII. [N]** When at least one the spouses is naturally sterile, the couple behaves like in Regimes VII and IX. The economic characteristics of couples living in Regime XII are heterogenous such that $c_{XII}^f \leq \hat{c}$.

### B.2 Optimal Decisions of Single Women

Conditional on being single, we can determine the optimal fertility and consumption of a woman as a function of her wage and non-labor income. We impose the following assumptions on the parameters, they ensures that the problem is not degenerated in the sense that only corner solutions prevail:

**Assumption 1**

$$1 - \delta f > \phi(\nu - \eta)(1 - \hat{c}), \quad \eta \geq \nu + \frac{\sqrt{1 - \delta f}(\sqrt{1 - \delta f} - 2\sqrt{\phi \nu})}{\phi}$$

We define non-labor income thresholds $a$ and $\overline{a}$, and wage thresholds, $W_0^f(a^f), W_1^f(a^f), W_2^f(a^f), W_3^f(a^f)$ and $W_4^f(a^f)$ as

---

31 Assumptions 1 and 2 (see below) hold under the values presented in Table 3.
Definition 1 (Wage thresholds for singles)

\[ W_f^0(a_f) = \frac{\hat{c} + \mu - a_f}{1 - \phi \eta - \delta_f}, \quad W_f^2(a_f) = \frac{2\hat{c} + \mu - a_f}{1 - \delta_f + \phi (\nu - \eta)}, \]

\[ W_f^3(a_f) = \frac{a_f - \mu}{1 - \delta_f + \phi (\nu - \eta)}, \quad W_f^5(a_f) = \frac{a_f - \mu}{\phi (\nu + \eta) + \delta_f - 1}, \]

\( W_f^1(a_f) \) is the smallest root, in \( w_f \), of the quadratic equation \( u(\hat{c}, n_{III}) = u(c_f^{IV}, 0) \):

\[
(1 - \delta_f)w_f + a_f - \mu \nu = \hat{c} \left( \frac{w_f(1 - \delta_f - \phi \eta) + a_f - \mu - \hat{c}}{\phi w_f} + \nu \right), \tag{8}
\]

\( W_f^4(a_f) \) is the highest root of the quadratic equation in \( w_f \):

\[
\frac{(w_f(1 - \delta_f + \phi (\nu - \eta)) + a_f - \mu)^2}{4\phi w_f} = \nu((1 - \delta_f)w_f + a_f - \mu).
\]

\( W_f^0(a_f) \) is the value of \( w_f \) above which a single woman can have children and consume at least \( \hat{c} \). \( W_f^2(a_f) \) is the value of \( w_f \) that solves \( n_{III} = n_i \). \( W_f^3(a_f) \) is the value of \( w_f \) such that \( n_i = n_M \). \( W_f^4(a_f) \) is either the lowest or the unique value of \( w_f \) such that \( u(c_f^{IV}, 0) = u(c_f^{I}, n_i) \). \( W_f^5(a_f) \) is the value of \( w_f \) that solves \( n_i = 0 \).

Definition 2 (Non-labor income thresholds for singles)

\[ a = \hat{c} \left( \frac{\phi (\nu + \eta) - 1 + \delta_f}{\phi \nu} \right) + \mu, \quad \bar{a} = \hat{c} + \mu > a \]

\( a \) is the value of \( a \) under which a single woman is always childless. \( \bar{a} \) is the value of \( a_f \) such that \( \hat{c} = c_f^{IV} \).

The following proposition describes the optimal behavior of a single woman endowed with \( (w_f, a_f) \).

Proposition 1 (Fertility of singles) Under Assumption 1, the optimal choice of a single woman who is not naturally sterile, and who is endowed with non-labor income \( a_f \) and wage \( w_f \), is given by:

1. when \( a_f < \bar{a} \):
   - \( \forall w_f < W_f^0(a_f), \ c_f = c_f^{II}, \ n = 0, \)
Proof. To prove Proposition 1, we need to show that, under Assumption 1:

1. when \( a^f < a \), \( W_5^f(a^f) < W_2^f(a^f) < W_0^f(a^f) \) and \( W_0^f(a^f) \) is greater than the highest root of Equation (8);

2. when \( a^f \in [a, \bar{a}] \), \( W_0^f(a^f) < W_1^f(a^f) < W_2^f(a^f) < W_4^f(a^f) < W_5^f(a^f) \);

3. when \( a^f \geq \bar{a} \), \( W_3^f(a^f) < W_4^f(a^f) < W_5^f(a^f) \).

Proof and Implications of 1.

Straightforward computations indicate that when \( a^f < a \), \( W_5^f(a^f) < W_2^f(a^f) < W_0^f(a^f) \). When \( w^f \leq W_0^f(a^f) \), becoming a mother is not feasible either in Regime I or III.

Once \( w^f > W_0^f(a^f) \), a single woman can have children in Regime III. Comparing utility in Regime IV with utility in Regime III, \( u(c_{IV}, 0) \leq u(\hat{c}, n_{II}) \) if and only if,

\[
(1 - \delta^f)w^f + a^f - \mu \leq \hat{c}\left(\frac{w^f(1 - \delta^f - \phi\eta) + a^f - \mu - \hat{c}}{\phi w^f} + \nu\right) .
\]

The left-hand side (hereafter LHS) is linear and increasing in \( w^f \) and the right-hand side (hereafter RHS) is increasing and concave in \( w^f \) (since \( a^f - \mu < \hat{c} \)), so Equation (9) holding with equality (i.e. Equation (8)) has at most two solutions. At \( W_0^f(a^f) \), the LHS of Equation (9) is higher than the RHS, implying that \( u(c_{IV}, 0) > u(\hat{c}, n_{II}) \). This implies that \( W_0^f(a^f) \)
is either smaller than the lowest root or greater than the highest root of Equation (8). Under Assumption 1, at \( W_2^f(a^f) \), \( u(\hat{c}, n_{iii}) \geq u(c^f_{IV}, 0) \) when \( a < \sigma \), meaning that \( W_2^f(a^f) \) is in between the roots of Equation (8). As \( W_0^f(a^f) > W_2^f(a^f), W_0^f(a^f) \) is above the highest root of Equation (8). We can then conclude that once the woman can have children in Regime I, choosing to live in this regime is not optimal (so we do not have to compare other indirect utilities to \( u(c^f_i, n_i) \)).

This means that when \( a^f < a \), women are either socially sterile [S], when \( w^f \leq W_0^f(a^f) \), or childless due to the high opportunity cost of having children [O], when \( w^f > W_0^f(a^f) \): once women become able to procreate (\( w^f > W_0^f(a^f) \)), becoming a mother is not optimal.

**Proof and Implications of 2.**

- First, we show that, for \( a^f \in [a, \bar{a}], W_0^f(a^f) < W_1^f(a^f) < W_2^f(a^f) \).

Considering Equation (9), when \( w^f = W_2^f(a^f) \), we have

\[
\text{LHS}(W_2^f(a^f)) = \frac{2(1 - \delta^f)\hat{c} + (a^f - \mu)\phi(\nu - \eta)}{1 - \delta^f + \phi(\nu - \eta)} \nu
\text{ and}

\[
\text{RHS}(W_2^f(a^f)) = \frac{\hat{c}^2}{2\hat{c} - a^f + \mu}\frac{1 - \delta^f + \phi(\nu - \eta)}{\phi}.
\]

When \( a^f \in [a, \bar{a}], \text{RHS}(W_2^f(a^f)) > \text{LHS}(W_2^f(a^f)) \) is satisfied under Assumption 1. This ensures that \( W_2^f(a^f) \) is in between the two roots that solve Equation (8). As \( W_1^f(a^f) \) is the smallest root of Equation (8) (Definition 1), \( W_1^f(a^f) < W_2^f(a^f) \).

\( W_0^f(a^f) < W_2^f(a^f) \) when \( a^f > a \) and the Inequality (9) is not satisfied for \( W_0^f(a^f) \). So \( W_0^f(a^f) < W_1^f(a^f) < W_2^f(a^f) \).

We can then conclude that, under Assumption 1, \( \forall w^f \in [W_0^f(a^f), W_1^f(a^f)], u(c^f_{IV}, 0) > u(\hat{c}, n_{iii}) \) and that \( \forall w^f \in [W_1^f(a^f), W_2^f(a^f)], u(\hat{c}, n_{iii}) \geq u(c^f_{IV}, 0) \).

- Second, we show that when \( a \in [a, \bar{a}], W_2^f(a^f) < W_4^f(a^f) < W_5^f(a^f) \).

At \( W_2^f(a^f) \), \( u(\hat{c}, n_{iii}) = u(c^f_i, n_i) \) and Regime I is feasible. So, \( \forall w^f > W_2^f(a^f), u(c^f_i, n_i) > u(\hat{c}, n_{iii}) \). At \( W_4^f(a^f) \), \( u(\hat{c}, n_{iii}) \geq u(c^f_{IV}, 0) \). So, we can conclude that \( u(c^f_i, n_i) \geq u(c^f_{IV}, 0) \) at \( W_2^f(a^f) \).

Regime I exists for \( w^f \in [W_2^f(a^f), W_4^f(a^f)] \). For all \( w^f > W_5^f(a^f), u(c^f_i, n_i) \) is not defined since \( \partial n_i / \partial w^f < 0 \), so \( n_i \) would be negative. Then, for \( w^f > W_5^f(a^f), \) Regime IV prevails. Let us compare the utility in Regime IV with the utility in Regime I when
\( w^f \leq \mathcal{W}_5^f(a^f) \). \( u(c_{IV}^f, 0) \geq u(c_i^f, n_i) \) if and only if,

\[
\ln \frac{(1 - \delta^f)w^f + a^f - \mu}{w^f(1 - \delta^f + \phi(\nu - \eta)) + a^f - \mu} + \ln \nu \geq \ln \frac{w^f(1 - \delta^f + \phi(\nu - \eta)) + a^f - \mu}{\phi w^f} - 2\ln 2.
\]

Considering Equation (10), at \( w^f = 0 \), the LHS is equal to \( \ln \nu \) and the RHS goes to \( +\infty \). The limits of the LHS and the RHS at \( +\infty \) are respectively,

\[
\ln \frac{1}{1 + \phi(\nu - \eta)} + \ln \nu \quad \text{and} \quad \ln \frac{1 + \phi(\nu - \eta)}{\phi} - 2\ln 2.
\]

This implies that the RHS is above the LHS for low values of \( w^f \) but as we cannot rank the two limits, the RHS can be above or below the LHS for large values of \( w^f \).

As both sides of the inequality are strictly decreasing and convex with \( w^f \), the LHS can be equal to the RHS for either one or two values of \( w^f \). At \( w^f = \mathcal{W}_5^f(a^f) \), the LHS is strictly larger than the RHS, implying that \( u(c_{IV}^f, 0) > u(c_i^f, n_i) \). This implies that \( \mathcal{W}_5^f(a^f) \) is either in between the roots of Equation (10) holding at equality or at the right of the only root. Since \( u(c_i^f, n_i) \) is not defined for \( w^f \geq \mathcal{W}_5^f(a^f) \), the relevant root of LHS=RHS, \( \mathcal{W}_4^f(a^f) \), is strictly lower than \( \mathcal{W}_5^f(a^f) \). This proves that \( u(c_{IV}^f, 0) < u(c_i^f, n_i) \) for \( w^f < \mathcal{W}_4^f(a^f) \), and that \( u(c_{IV}^f, 0) \geq u(c_i^f, n_i) \) for \( w^f \in [\mathcal{W}_4^f(a^f), \mathcal{W}_5^f(a^f)](a^f) \).

Intuitively, because there is a fixed cost to becoming a parent, the optimal fertility is not continuous in \( w^f \).

We showed that \( \forall w^f < \mathcal{W}_4^f(a^f), \; u(c_i^f, n_i) > u(c_{IV}^f, 0) \) and that at \( \mathcal{W}_4^f(a^f) \), Regime I exists and \( u(c_i^f, n_i) > u(c_{IV}^f, 0) \). This implies that \( \mathcal{W}_2^f(a^f) < \mathcal{W}_4^f(a^f) \).

Regime V is not reachable when \( a^f < \bar{a} \) since the consumption in Regime V is equal to \( a^f - \mu \), which does not allow the woman to reach the minimal consumption level allowing her to procreate.

We have proved that, when \( a \in [\underline{a}, \bar{a}], \; \mathcal{W}_0^f(a^f) < \mathcal{W}_1^f(a^f) < \mathcal{W}_2^f(a^f) < \mathcal{W}_3^f(a^f) < \mathcal{W}_5^f(a^f) \) under Assumption 1. We can then conclude that when \( a \in [\underline{a}, \bar{a}] \),

- \( \forall w^f < \mathcal{W}_0^f(a^f), \; c^f = c_{IV}^f, \; n = 0 \),
- \( \forall w^f \in [\mathcal{W}_0^f(a^f), \mathcal{W}_1^f(a^f)], \; u(c_{IV}^f, 0) > u(\hat{c}, n_{III}) \) and \( c^f = c_{IV}^f, \; n = 0 \),
- \( \forall w^f \in [\mathcal{W}_1^f(a^f), \mathcal{W}_2^f(a^f)], \; u(\hat{c}, n_{III}) \geq u(c_{IV}^f, 0) \) and \( c^f = \hat{c}, \; n = n_{III} \),
- \( \forall w^f \in [\mathcal{W}_2^f(a^f), \mathcal{W}_3^f(a^f)], \; u(c_i^f, n_i) > u(c_{IV}^f, 0) \) and \( c^f = c_i^f, \; n = n_i \),
- \( \forall w^f \geq \mathcal{W}_3^f(a^f), \; c^f = c_{IV}^f, \; n = 0 \).
Proof and Implications of 3.

We are now in the case \( a^f \geq \bar{a} \), where Regime V is feasible for all \( w^f \geq 0 \). Furthermore, Regimes II and III no longer exist since even with a wage equal to zero, a woman can consume more than \( \hat{c} \). We then just need to compare the utilities in Regimes I, IV and V.

We can check that \( n_i > n_M \iff w^f < \mathcal{W}_4^I(a^f) \). \( \forall w^f < \mathcal{W}_4^I(a^f), u(c^f_i, n_i) \) is not defined as \( n_i \) would be above the possible maximum.

Let us now show that \( \mathcal{W}_3^I(a^f) < \mathcal{W}_4^I(a^f) < \mathcal{W}_5^I(a^f) \). We already know that both the LHS and the RHS of Equation (10) are strictly decreasing and convex in \( w^f \). We can check that at \( \mathcal{W}_3^I(a^f) \) the LHS is lower than the RHS. It follows from Definition 1 that \( \mathcal{W}_3^I(a^f) < \mathcal{W}_5^I(a^f) \).

Since at \( \mathcal{W}_4^I(a^f) \) the LHS is equal to the RHS, then \( \mathcal{W}_3^I(a^f) < \mathcal{W}_4^I(a^f) < \mathcal{W}_5^I(a^f) \).

At \( \mathcal{W}_3^I(a^f) \), we can show that \( u(c^f_i, n_M) = u(c^f_i, n_i) > u(c^f_{iv}, 0) \). As \( u(c^f_i, n_i) \) is increasing in \( w^f \) and \( u(c^f_i, n_M) \) is unaffected by \( w^f \), we have that \( u(c^f_i, n_i) \geq u(c^f_i, n_M) \) for \( w^f \in [\mathcal{W}_3^I(a^f), \mathcal{W}_5^I(a^f)] \). As \( u(c^f_{iv}, 0) \) is increasing in \( w^f \) and \( u(c^f_i, n_M) \) is unaffected by \( w^f \), we also have that \( u(c^f_i, n_M) > u(c^f_{iv}, 0) \forall w^f < \mathcal{W}_3^I(a^f) \).

Given that \( \mathcal{W}_3^I(a^f) < \mathcal{W}_4^I(a^f) < \mathcal{W}_5^I(a^f) \), we can conclude that:

- when \( w^f < \mathcal{W}_3^I(a^f) \), \( c^f = c^{iv} \) and \( n = n_M \);
- when \( w^f \in [\mathcal{W}_3^I(a^f), \mathcal{W}_4^I(a^f)] \), \( u(c^f_{iv}, 0) \geq u(c^f_i, n_i) > u(c^f_i, n_M) \), which implies that \( c^f = c^f_i \) and \( n = n_i \);
- when \( w^f > \mathcal{W}_4^I(a^f) \), \( u(c^f_{iv}, 0) > u(c^f_i, n_M) \) and \( c^f = c_{iv}^f \) and \( n = 0 \).

The following paragraphs describe the main results of this proposition.

Figure 19 represents the relationship between the fertility of a single fecund woman and her wage, when \( a^f \geq \bar{a} \). The left panel shows a non-monotonic relationship between wage and fertility. The net non-labor income \( (a^f - \mu) \) is still not large enough to cover alone the minimal amount of consumption needed to procreate. For a low \( w^f \), Regime III prevails and an increase in the wage of the woman increases her fertility. As her consumption is fixed at level \( \hat{c} \) in this regime, fertility is positively related to her wage until it is high enough to reach Regime I. In Regime I, \( n_i \) decreases with \( w^f \) because the substitution effect dominates \( (a^f \geq \bar{a} > \mu) \).

In the right panel, non-labor income is high enough to ensure a consumption \( \hat{c} \) for any fertility level, even in the absence of labor income. As a consequence, Regime III disappears, and
Regime V becomes possible. Regime V is a corner solution where the wage of the woman is so small that it is optimal for her to specialize in childrearing and consume her net non-labor income \( a' - \mu \).

On the whole, Figure 19 shows that uneducated single women will either be socially sterile or in the maximum fertility case, this will depend on their non-labor income. Conversely, highly educated women will be childless (Regime IV) because the opportunity cost of having children (non-participation to the workforce) is high. This figure also shows another feature: the existence of a fixed cost of becoming a parent implies that the passage from childlessness to parenthood is not represented by a continuous function. The fixed cost makes it never optimal to have a very low number of children.

### B.3 Optimal Decisions of Couples

We now describe the optimal decisions of couples and show that for some values of the model’s parameters, the relationship between fertility and the wage of women is rightly described in Section 3. Notice that all the cases described in this section exist for \( w^m > 0 \) but sufficiently small to ensure that all the thresholds on \( a \) are positive and for \( \theta(w^f, w^m) \) fixed to \( \hat{\theta} \). At the end of this subsection, we show that our proof still holds in the cases where \( w^m \) becomes high. Indeed, in these cases, situations of poverty disappear and things become much easier. We also evoke how our results are changed when \( \theta \) is not fixed but depends on \( w^f \) and \( w^m \).

Definitions for wage and non-labor income thresholds are given in Definitions 3 and 4 respectively. These thresholds are used to describe the optimal decisions of couples as a function of wages and non-labor incomes, formally stated in Proposition 2. Results are valid under Assumption 2.
Definition 3 (Wage thresholds for couples)

\[ W_{A}(a, w^m) = \frac{\hat{c} - a - (1 - \phi(1 - \alpha)\eta)w^m}{1 - \alpha\phi\eta}, \]
\[ W_{B}(a, w^m) = \frac{\hat{c} - a + (\phi(1 - \alpha)((1 - \hat{\theta})\nu - \eta) - 1)w^m}{1 - \alpha\phi(\eta + \nu(1 - \hat{\theta}))}, \]
\[ W_{C}(a, w^m) = \frac{\Xi_1 + \left(\Xi_1^2 + 4\hat{\theta}\alpha(1 + \phi\alpha(\nu - \eta))\Xi_2\right)^{\frac{1}{2}}}{2\hat{\theta}\alpha(1 + \alpha\phi(\nu - \eta))}, \]
\[ W_{D}(a, w^m) = \frac{\Xi + (\Xi^2 + 4\hat{\theta}\alpha(1 + \phi\alpha(\nu - \eta)))\Xi_2}{2\hat{\theta}\alpha(1 + \alpha\phi(\nu - \eta))}, \]
\[ W_{E}(a, w^m) = \frac{\Xi + (\Xi^2 + 4\hat{\theta}\alpha(1 + \phi\alpha(\nu - \eta)))\Xi_2}{2\hat{\theta}\alpha(1 + \alpha\phi(\nu - \eta))}, \]
\[ W_{F}(a, w^m) = \frac{\Xi + (\Xi^2 + 4\hat{\theta}\alpha(1 + \phi\alpha(\nu - \eta)))\Xi_2}{2\hat{\theta}\alpha(1 + \alpha\phi(\nu - \eta))}, \]
\[ W_{G}(a, w^m) = \frac{\Xi + (\Xi^2 + 4\hat{\theta}\alpha(1 + \phi\alpha(\nu - \eta)))\Xi_2}{2\hat{\theta}\alpha(1 + \alpha\phi(\nu - \eta))}, \]
\[ W_{H}(a, w^m) = \frac{\Xi + (\Xi^2 + 4\hat{\theta}\alpha(1 + \phi\alpha(\nu - \eta)))\Xi_2}{2\hat{\theta}\alpha(1 + \alpha\phi(\nu - \eta))}, \]
\[ W_{I}(a, w^m) = \frac{\Xi + (\Xi^2 + 4\hat{\theta}\alpha(1 + \phi\alpha(\nu - \eta)))\Xi_2}{2\hat{\theta}\alpha(1 + \alpha\phi(\nu - \eta))}, \]
\[ W_{J}(a, w^m) = \frac{\Xi + (\Xi^2 + 4\hat{\theta}\alpha(1 + \phi\alpha(\nu - \eta)))\Xi_2}{2\hat{\theta}\alpha(1 + \alpha\phi(\nu - \eta))}, \]

where \( \Xi_1 \equiv (1 - \hat{\theta})(1 + \phi\alpha(\nu - \eta))(w^m + \alpha a) - 2\alpha(a - \hat{c} + (1 + \phi(1 - \alpha)(\nu - \eta))w^m), \)
\[ \Xi_2 \equiv 2(1 - \hat{\theta})(1 + \phi\alpha(\nu - \eta))(1 - \alpha)w^m(a + w^m) - (a + \alpha w^m)(a - \hat{c} + (1 + \phi(1 - \alpha)(\nu - \eta))w^m), \]
\[ \Xi_3 \equiv 2(\alpha w^m[1 - \phi(\nu + \eta) + \phi\alpha(1 - \alpha)(\nu - \eta)^2] + a[1 - \phi\alpha(\nu + \eta)]), \]
\[ \Xi_4 \equiv (a + (1 + \phi(1 - \alpha)[\nu - \eta])w^m - a[1 - \phi\alpha(\nu + \eta)]), \]

and \( \Xi_4 \equiv (a + (1 + \phi(1 - \alpha)[\nu - \eta])w^m - a[1 - \phi\alpha(\nu + \eta)]), \)

We finally denote as \( W_{C}(a, w^m) \) the two potential roots of the equation

\[ U(\hat{c}, c_{\text{viii}}^m, n_{\text{vii}}) = U(c_{\text{ix}}^m, c_{\text{ix}}^m, 0) \]
\(W_f(a, w^m)\) is such that the couple can afford positive fertility \((n \to 0, c^f = \hat{c} \text{ and } c^m = 0)\). \(W_f(a, w^m)\) is such that \(n_{vIII} = 0\). \(W_C(a, w^m)\) is the value of \(w^f\) that maximizes \(U(c^m_{vIII}, c^m_{vIII}, n_{vIII}) - U(c^m_{vIII}, c^m_{vIII}, 0)\). \(W_D(a, w^m)\) is such that \(n_{vIII} = n_{vI}\). \(W_E(a, w^m)\) is such that \(n_{vIII} = \bar{n}_M\). \(W_F\) is such that \(n_{vI} = \bar{n}_M\). \(W_G\) is the unique root of \(U(c^m_{vIII}, c^m_{vIII}, \bar{n}_M) - U(c^m_{vIII}, c^m_{vIII}, 0) = 0\). \(W_I(a, w^m)\) is such that \(U(c^m_{vIII}, c^m_{vIII}, n_{vI}) = U(c^m_{vIII}, c^m_{vIII}, 0)\) while \(W_2(a, w^m)\) and \(W_J(a, w^m)\) are the lowest and the highest roots of the equation \(U(c^m_{vIII}, c^m_{vIII}, n_{vI}) = U(c^m_{vIII}, c^m_{vIII}, 0)\) respectively.

For ease of notation, we will denote \(W_k(a, w^m)\) (for \(k = A, B, C, D, E, F, G, I, J, \bar{J}\)) as \(W_k\) and use the complete notation only when necessary.

**Definition 4 (Non-labor income thresholds for couples)**

\(A_0\) is the value of \(a\) that solves \(U(\hat{c}, c^m_{vIII}, n_{vIII}) = U(c^f_{vIII}, c^m_{vIII}, 0)\) when \(w^f = W_f\).

\(A_1\) is the value of \(a\) that solves \(U(c^f_{vIII}, c^m_{vIII}, n_{vIII}) = U(c^f_{vIII}, c^m_{vIII}, 0)\) when \(w^f = W_D\).

\(A_2 = \hat{c} - (1 - \phi (1 - \alpha))\) is the value of \(a\) above which a couple can have children even if the wife has no labor income (or has the maximal number of children).

\(A_3\) is the lowest value of \(a\) such that \(W_G = 0\).

\(A_4\) is the value of \(a\) such that \(W_f = W_E\).

\(A_5 = \frac{2\hat{c}}{\theta} - \frac{2\alpha - 1}{\alpha} w^m\). Above \(A_5\), even when her wage is zero and her negotiation power is minimal, a woman can consume more than \(\hat{c}\) even in the interior regime.

**Assumption 2** Parameters \(\alpha, \phi, \nu, \eta, \theta\), and \(\hat{c}\) satisfy:

- \((1 + \alpha \phi (\nu - \eta))^2 - 4\phi \alpha \nu < 0\), this condition ensures that the problem is not degenerated, in the sense that the interior regime is chosen for some values of \(w^f\) and \(a\),

- when \(w^f = W_D\),

\[
\frac{\partial U(\hat{c}, c^m_{vIII}, n_{vIII})}{\partial w^f} = \frac{\partial U(c^f_{vIII}, c^m_{vIII}, n_{vIII})}{\partial w^f} - \varepsilon
\]

with \(\varepsilon \to 0^+\), this assumption excludes a degenerated case where, over a range of \(w^f\), couples first choose the Regime VIII then Regime IX and then Regime VI.

- when \(w^f = W_C\),

\[
\frac{\partial}{\partial a} \left[U(\hat{c}, c^m_{vIII}, n_{vIII}) - U(c^f_{vIII}, c^m_{vIII}, n_{vIII})\right] > 0, \forall a < A_4.
\]

This assumption ensures that Regime VIII will be chosen for some triplets of \((w^f, w^m, a)\).
Proposition 2  Under Assumption 2, the optimal choice of a couple with non-labor income $a$ and wages $w^f$ and $w^m$ is given by:

1. When $a \leq A_0$:
   - if $w^f < W^f_A$, $c^f = c^f_{VII}$, $c^m = c^m_{VII}$, $n = 0$
   - if $w^f \geq W^f_A$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$

2. When $a \in ]A_0, A_1[$:
   - if $w^f < W^f_A$, $c^f = c^f_{VII}$, $c^m = c^m_{VII}$, $n = 0$
   - if $w^f \in ]W^f_A, W^f_C[$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$
   - if $w^f \in ]W^f_C, W^f_D[$, $c^f = c^f_{VIII}$, $c^m = c^m_{VIII}$, $n = n_{VIII}$
   - if $w^f > W^f_D$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$

3. When $a \in ]A_1, A_2[$:
   - if $w^f < W^f_A$, $c^f = c^f_{VII}$, $c^m = c^m_{VII}$, $n = 0$
   - if $w^f \in ]W^f_A, W^f_C[$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$
   - if $w^f \in ]W^f_C, W^f_D[$, $c^f = c^f_{VIII}$, $c^m = c^m_{VIII}$, $n = n_{VIII}$
   - if $w^f \in ]W^f_D, W^f_I[$, $c^f = c^f_{VII}$, $c^m = c^m_{VII}$, $n = n_{VII}$
   - if $w^f > W^f_I$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$

4. When $a \in ]A_2, A_3[$:
   - if $w^f < W^f_C$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$
   - if $w^f \in ]W^f_C, W^f_D[$, $c^f = c^f_{VIII}$, $c^m = c^m_{VIII}$, $n = n_{VIII}$
   - if $w^f \in ]W^f_D, W^f_I[$, $c^f = c^f_{VII}$, $c^m = c^m_{VII}$, $n = n_{VII}$
   - if $w^f > W^f_I$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$

5. When $a \in ]A_3, A_4[$:
Step 1: Some General Properties of the Model

We use a two-step strategy. First, we highlight some general properties of the model. Second, we prove each part of the proposition. Notice that all the cases exist only when $w^m$ is sufficiently close to zero. Successive increases of $w^m$ will make cases of poverty disappear. For instance, one can easily check that for high values of $w^m$, $W_A$, $W_B$, $A_2$ and $A_5$ become negative making the study of fertility behaviors of couples much easier.

**Proof.** We use a two-step strategy. First, we highlight some general properties of the model.

In what follows, we assume $w^m$ small enough to ensure that $A_0 > 0$.

- **Regimes VII and IX** are “continuous” in the sense that $U(c_{vIII}^l, c_{vIII}^m, 0) = U(c_{IX}^l, c_{IX}^m, 0)$, $\forall w^f > 0$. A couple is able to have children once its total income is greater than $\hat{c}$ which is satisfied when $w^f > W_A$. This implies that $\forall w^f < W_A$, childlessness is due to poverty, while it is due to a high opportunity cost of having children otherwise.

- **Regimes VIII and VI** are “continuous”. Regime VIII exists as long as $n_{vIII} \geq 0$ which is satisfied $\forall w^f \geq W_B$. Regime VI exists once $c_{VI}^l \geq \hat{c}$ which is satisfied $\forall w^f \geq W_D$. We can verify that when $w^f = W_B$, $c_{VI}^l = \hat{c}$ and $n_{VI} = n_{vIII}$, implying that $U(\hat{c}, c_{vIII}^m, n_{vIII}) = U(c_{vIII}^l, c_{vIII}^m, n_{vIII})$. It also implies that Regime VIII is defined $\forall w^f \in [W_B, W_D]$ while Regime VI is defined $\forall w^f \geq W_D$ ($\lim_{w^f \to \infty} n_{VI} > 0$).
• Equation $U(c_{vI}^f, c_{vI}^m, n_{vI}) = U(c_{ix}^f, c_{ix}^m, 0)$ can be written as

\[
([1 + \alpha \phi (\nu - \eta)^2] - 4\phi \alpha \nu) (w_f)^2 - \Xi_3 w_f + \Xi_4 = 0. \tag{11}
\]

As $[1 + \alpha \phi (\nu - \eta)^2] - 4\phi \alpha \nu < 0$, Equation (11) has, at most, two positive roots and admits a unique maximum at $w_f = \mathcal{W}_f^f$, then $\mathcal{W}_f^f > 0$ if $\Xi_3 > 0$.

• When $a^f \leq A_0$:
  
  i. $\forall w^f > 0$, $U(c_{vI}^f, c_{vI}^m, 0) = U(c_{ix}^f, c_{ix}^m, 0) > U(\hat{c}, c_{vIII}^m, n_{vIII})$ and,

  ii. when $w^f = \mathcal{W}_D^f$, $U(c_{vI}^f, c_{vI}^m, n_{vI}) = U(\hat{c}, c_{vIII}^m, n_{vIII}) < U(c_{ix}^f, c_{ix}^m, 0)$.

$U(\hat{c}, c_{vIII}^m, n_{vIII}) - U(c_{ix}^f, c_{ix}^m, 0)$ is positive when,

\[
(1 + \phi \alpha (\nu - \eta))w^f + (1 + \phi (1 - \alpha)(\nu - \eta))w^m + a - \hat{c} \geq \left(\frac{\hat{\theta}}{\hat{c}}\right)^\frac{\alpha}{2-\alpha} (2 - \hat{\theta}) \left[ \nu \phi \alpha (w^f + w^m + a)(\alpha w^f + (1 - \alpha)w^m) \right]^\frac{1}{2-\alpha}. \tag{12}
\]

Since the LHS is linearly increasing with respect to $w^f$ and the RHS is increasing and convex with $w^f$, we know that $U(\hat{c}, c_{vIII}^m, n_{vIII}) - U(c_{ix}^f, c_{ix}^m, 0)$ admits a unique maximum, denoted $\mathcal{W}_C^f$, and at most two positive roots, denoted $\mathcal{W}_C^f < \mathcal{W}_C^f$. Furthermore, we can check that Inequality (12) is not satisfied when $w^f = 0$ and $a < \hat{c} - (1 + \phi (1 - \alpha)(\nu - \eta)w^m)$.

Under Assumption 2, when $w^f = \mathcal{W}_C^f$, $U(\hat{c}, c_{vIII}^m, n_{vIII}) - U(c_{ix}^f, c_{ix}^m, 0)$ is increasing in $a$, for $a < A_4$. This means that the maximum value of $U(\hat{c}, c_{vIII}^m, n_{vIII}) - U(c_{ix}^f, c_{ix}^m, 0)$ increases monotonically with $a$. It implies that there is a value, $a = A_0$, where $A_0$ solves

\[
(1 + \phi \alpha (\nu - \eta))\mathcal{W}_C^f(A_0, w^m) + (1 + \phi (1 - \alpha)(\nu - \eta))w^m + A_0 - \hat{c} = \left(\frac{\hat{\theta}}{\hat{c}}\right)^\frac{\alpha}{2-\alpha} (2 - \hat{\theta}) \left[ \nu \phi \alpha (\mathcal{W}_C^f(A_0, w^m) + w^m + A_0)(\alpha \mathcal{W}_C^f(A_0, w^m) + (1 - \alpha)w^m) \right]^\frac{1}{2-\alpha}. \tag{13}
\]

This means that when $a^f \leq A_0$, $U(c_{vI}^f, c_{vI}^m, 0) = U(c_{ix}^f, c_{ix}^m, 0) > U(\hat{c}, c_{vIII}^m, n_{vIII}), \forall w^f \geq 0$. As $w^f = \mathcal{W}_D^f$ implies that $U(c_{vI}^f, c_{vI}^m, n_{vI}) = U(c_{vIII}^f, c_{vIII}^m, n_{vIII})$, we also find that $U(c_{vI}^f, c_{vI}^m, n_{vI}) < U(c_{ix}^f, c_{ix}^m, n_{ix})$ for $w^f = \mathcal{W}_D^f$ and $a \leq A_0$. 

62
• \( U(c_{Vt}^f, c_{Vt}^m, n_{Vt}) < U(c_{IX}^f, c_{IX}^m, 0), \forall w^f > W_D^f \) and \( a \leq A_1 \). As already shown, Equation (11) is quadratic with respect to \( w^f \). It admits a unique maximum for \( w^f = W_J^f < W_D^f \) when \( a \leq A_1 \). Then \( U(c_{Vt}^f, c_{Vt}^m, n_{Vt}) - U(c_{IX}^f, c_{IX}^m, 0) \) is decreasing \( \forall w^f > W_D^f \). Since \( \forall a \leq A_1 \) and \( w^f = W_D^f, U(c_{IX}^m, c_{IX}^m, 0) > U(c_{VIII}^m, c_{VIII}^m, n_{VIII}) = U(c_{Vt}^m, c_{Vt}^m, n_{Vt}) \), then \( \forall a \leq A_1 \) and \( \forall w^f > W_D^f, U(c_{Vt}^m, c_{Vt}^m, n_{Vt}) < U(c_{IX}^m, c_{IX}^m, 0) \).

• Regime X is never preferred to Regime IX when \( a < A_3 \). This is due to \( W_G^f < 0 \) for \( a < A_3 \).

• There exists a unique \( A_4 > 0 \) such that \( \forall a \in [A_3, A_4], W_G^f \leq W_E^f \) while \( \forall a \in [A_4, A_5] \), \( W_G^f > W_E^f \). Equation \( W_G^f = W_E^f \) can be written as

\[
\left( \frac{\hat{c}}{\hat{\theta}} \right) \theta \left( \frac{2a-1}{\alpha} w^m + a - \hat{c} \right)^{1-\hat{\theta}} \frac{1 + \alpha \phi (\nu - \eta)}{\alpha \phi \nu} = a + w^m - \frac{a - \hat{c} + \left( 1 - (1 - \hat{\theta}) \phi(1 - \alpha)(\nu - \eta) - \frac{1 - \alpha}{\alpha} (2 - \hat{\theta}) \right) w^m}{(1 - \hat{\theta})(1 + \alpha \phi (\nu - \eta))} \tag{14}
\]

where the LHS is increasing and concave with respect to \( a \) while the RHS is linearly increasing with \( a \). It implies that the equation has at most two roots and as the LHS is only defined for \( a > \hat{c} - \frac{2a-1}{\alpha} w^m \), which is positive for sufficiently low values of \( w^m \), these roots are strictly positive. Furthermore, under Assumption 2, \( W_G^f(A_2, w^m) < W_E^f(A_2, w^m) \) and \( A_5 \) lies in between the two roots of Equation (14). It directly implies that there exists a unique \( A_4 \) such that \( W_G^f < W_E^f \) when \( a \in [A_3, A_4[ \) and \( W_G^f \geq W_E^f \) \( \forall a \in [A_4, A_5] \).

**Step 2: Proof of Each Part of Proposition 2**

We still assume \( w^m \) low enough for \( A_0 \) to be positive. This will be relaxed afterwards.

**Step 2.1: Cases where \( a < A_2 \)**

Regimes X and XI are not reachable because \( c^m_X < 0 \) and \( c^f_{XI} < \hat{c} \) for \( a^f < A_2 \). So we only need to compare the utilities \( U_{VII}(a, w^f), U_{VIII}(a, w^f), U_{VI}(a, w^f) \) and \( U_{IX}(a, w^f) \) to see the outcome of a marriage.

**Case 1: \( a < A_0 \):** Here, we show that, as in the first case of Proposition 1, once agents with very low non-labor income are able to be parents, the opportunity cost of childrearing is too high for them to be willing to. \( W_A^f \), defined in Definition 3, solves:

\[
(1 - \phi \alpha \eta) W_A^f + 1 - \phi (1 - \alpha) \eta w^m + a = \hat{c}.
\]
We can check that \( \mathcal{W}^f_A > 0 \Leftrightarrow a < A_2 \) and that \( A_0 < A_2 \). For \( w^f < \mathcal{W}^f_A \), the consumption of a woman who wants to have children would be lower than \( \hat{c} \) even if her husband does not consume. Consequently, for \( w^f < \mathcal{W}^f_A \), the couple is socially sterile. For \( w^f \geq \mathcal{W}^f_A \), the couple can decide whether to have children or to be childless in Regime IX.

When \( a < A_0 \), \( U(c_{\text{viii}}, c_{\text{viii}}^m, n_{\text{viii}}) < U(c_{\text{ix}}, c_{\text{ix}}^m, 0) \), \( \forall w^f \). By continuity, we know that \( U(c_{\text{viii}}, c_{\text{viii}}^m, n_{\text{viii}}) \) is lower than \( U(c_{\text{ix}}, c_{\text{ix}}^m, 0) \) for \( w^f = \mathcal{W}^f_D \). As shown in Step 1 of the proof, when \( a < A_1 \), \( U(c_{\text{viii}}, c_{\text{viii}}^m, n_{\text{viii}}) \) decreases with \( w^f \), which directly implies that \( \forall w^f > \mathcal{W}^f_D \), \( U(c_{\text{viii}}, c_{\text{viii}}^m, n_{\text{viii}}) < U(c_{\text{ix}}, c_{\text{ix}}^m, 0) \).

We can then conclude that if \( w^f < \mathcal{W}^f_A \), \( c^f = c_{\text{viii}}^f \), \( c^m = c_{\text{viii}}^m \), \( n = 0 \), while if \( w^f \geq \mathcal{W}^f_A \), \( c^f = c_{\text{ix}}^f \), \( c^m = c_{\text{ix}}^m \), \( n = 0 \).

**Case 2:** \( a \in [A_0, A_1] \): In this case, we will show that Malthusian mechanisms, where fertility is related positively to female’s wages, can be at play among some poor couples.

As \( A_1 < A_2 \), \( \mathcal{W}^f_A > 0 \) and couples are socially sterile when \( w^f < \mathcal{W}^f_A \). Furthermore,

\[
\lim_{w^f \to \mathcal{W}^f_A} U(c_{\text{viii}}^f, c_{\text{viii}}^m, n_{\text{viii}}) = -\infty
\]

because \( c_{\text{viii}}^m \to 0 \) when \( w^f \to \mathcal{W}^f_A \). \( U(c_{\text{viii}}^f, c_{\text{viii}}^m, n_{\text{viii}}) \) is then lower than \( U(c_{\text{ix}}^f, c_{\text{ix}}^m, 0) \) in the neighborhood of \( \mathcal{W}^f_A \).

When \( a > A_0 \), the difference \( U(c_{\text{viii}}^f, c_{\text{viii}}^m, n_{\text{viii}}) - U(c_{\text{ix}}^f, c_{\text{ix}}^m, 0) \) admits two positive roots, denoted \( \mathcal{W}^f_C \) and \( \mathcal{W}^f_D \), and is positive between them. As \( a < A_1 \), when \( w^f \geq \mathcal{W}^f_D \), \( U(c_{\text{viii}}^f, c_{\text{viii}}^m, n_{\text{viii}}) = U(c_{\text{viii}}^f, c_{\text{viii}}^m, n_{\text{viii}}) < U(c_{\text{viii}}^f, c_{\text{viii}}^m, 0) \). This means that once the unconstrained fertility regime can be reached by the couple, having children is no longer optimal.

We conclude that for \( a \in [A_0, A_1] \), couples are in Regime VII for \( w^f < \mathcal{W}^f_A \), in Regime IX for \( w^f \in ]\mathcal{W}^f_A, \mathcal{W}^f_C[ \), in Regime VIII for \( w^f \in ]\mathcal{W}^f_C, \mathcal{W}^f_D[ \) and in Regime IX again for \( w^f > \mathcal{W}^f_D \).

**Case 3:** \( a \in [A_1, A_2] \): In this case, we can show that for intermediary values of non-labor incomes, a couple can either be in a Malthusian or a modern fertility pattern, where fertility decreases with the mothers’ wage.

As \( a \leq A_2 \), \( \mathcal{W}^f_A \geq 0 \) and couples are socially sterile for \( w^f < \mathcal{W}^f_A \). Using the same reasoning as for the previous case,

\[
\lim_{w^f \to \mathcal{W}^f_A} U(c_{\text{viii}}^f, c_{\text{viii}}^m, n_{\text{viii}}) = -\infty
\]

and \( U(c_{\text{viii}}^f, c_{\text{viii}}^m, n_{\text{viii}}) < U(c_{\text{ix}}^f, c_{\text{ix}}^m, 0) \) in the neighborhood of \( \mathcal{W}^f_A \).
Since \( a > A_1 \), \( U(c_{v_1}^f, c_{v_1}^m, n_{v_1}) \) is higher than \( U(c_{v_3}^f, c_{v_3}^m, 0) \) for \( w^f = W_D^f \). \( W_D^f \) then lies in-between the roots of Equation (11), denoted by \( W_J^f \) and \( W_
abla^f \). Furthermore, as \( U(c_{v_3}^f, c_{v_3}^m, n_{v_3}) = U(c_{v_1}^f, c_{v_1}^m, n_{v_1}) \) for \( w^f = W_D^f \), we know that \( W_D^f \) also lies in between \( W_
abla^f \) and \( W_J^f \).

We conclude that for \( a \in]A_1, A_2[ \), couples are in Regime VII when \( w^f < W_A^f \), in Regime IX when \( w^f \in]W_A^f, W_J^f[ \), in Regime VIII when \( w^f \in]W_
abla^f, W_D^f[ \), in Regime VI when \( w^f \in ]W_D^f, W_J^f[ \) and in Regime IX again when \( w^f > W_
abla^f \).

**Step 2.2: Cases where \( a \geq A_2 = \hat{c} \)**

For \( a > A_2 \), Regime VII no longer exists: a couple can procreate for any \( w^f \geq 0 \). Regimes X and XI become reachable.

**Case 4: \( a \in]A_2, A_3[ \)**:

We will show that in this case, having the maximum fertility is not yet optimal.

Here, \( W_G^f < 0 \). As long as \( w^f < W_D^f \), only Regimes IX, VIII and X can be reached. We then have to compare Regimes VIII and IX for \( w^f < W_D^f \). As \( a > A_0, U(c_{v_3}^f, c_{v_3}^m, n_{v_3}) \geq U(c_{v_1}^f, c_{v_1}^m, 0) \) for \( w^f \in ]W_
abla^f, W_D^f[ \). This implies that Regime IX prevails \( \forall w^f \in ]0, W_
abla^f[ \).

As shown above, if \( a > A_1 \), then \( W_D^f \in [\min\{W_G^f, W_J^f\}, \max\{W_G^f, W_J^f\}] \). This implies that Regime VIII prevails \( \forall w^f \in ]W_G^f, W_D^f[ \) while Regime VI prevails \( \forall w^f \in ]W_D^f, W_J^f[ \). Finally, Regime IX prevails when \( w^f \geq W_J^f \).

**Case 5: \( a \in]A_3, A_4[ \)**:

For these values of \( a \), having the maximum number of children is feasible only in Regime X. This behavior will be optimal only for very low values of \( w^f \).

In line with previous cases and Step 1, under Assumption 2, we know that, for \( a \in]A_3, A_4[ \), \( W_G^f < \max\{W_G^f, W_E^f\} < W_D^f < W_J^f \). We can conclude that Regime X prevails when \( w^f < W_G^f \) while Regime IX prevails when \( w^f \in ]W_G^f, \max\{W_G^f, W_E^f\}[ \). When \( w^f \in ]\max\{W_G^f, W_E^f\}, W_D^f[ \), Regime VIII is chosen by the couple while Regime VI is preferred to others when \( w^f \in ]W_D^f, W_J^f[ \). Finally, for \( w^f > W_J^f \), Regime IX prevails again.

**Case 6: \( a \in]A_4, A_5[ \)**: The only difference between Case 6 and Case 5 lies in the fact that for \( a \in]A_4, A_5[ \), \( W_G^f > W_E^f \). Therefore, we have that \( W_E^f < W_G^f < W_D^f < W_J^f \). The results of Case 6 directly follow.

**Case 7: \( a > A_5 \)**: When \( a > A_5 \), the constrained regimes \([C] \) (Regime VIII and X) no longer exist. Indeed, a woman with a wage equal to zero (or equivalently, having the maximum number of children) would consume more than \( \hat{c} \). This implies that only Regimes VI, IX
and XI potentially exist. Under Assumption 2, \( W_F^f < W_f^I < W_J^f \). One can easily check that \( W_F^f(a, 0) < W_f^I(a, 0) \); intuitively, higher \( w_m \) will reinforce this inequality as higher \( w_m \) means a smaller opportunity cost for the wife to fully specialize in labor market activities.

For \( w_f^I < W_f^I \), \( U(c_f^{X}, c_{m}^{X}, n_{X}) > U(c_f^{I}, c_{m}^{I}, n_{I}) \). By continuity, we know that for \( w_f^I = W_f^I \), \( U(c_f^{I}, c_{m}^{I}, n_{I}) > U(c_f^{I}, c_{m}^{I}, n_{I}) \) (which is why we know that \( W_f^I < W_J^f \)). This implies that for \( w_f^I \in [W_f^I, W_J^f] \), Regime VI prevails, while for \( w_f^I > W_J^f \), Regime IX prevails.

\[ \]  

Relaxing assumptions on \( w_m \) and \( \theta \):

As explained at the beginning of the proof, all the seven cases we have studied exist for \( \theta(w_f^I, w_m^I) \) fixed to \( \hat{\theta} \) and when \( w_m^I > 0 \) but sufficiently small to ensure that all the thresholds on \( a \) are positive. Keeping \( \theta(w_f^I, w_m^I) = \hat{\theta} \), successive increases of \( w_m^I \) will make some cases disappear. For instance, if a couple enjoy a \( w_m^I > \hat{\theta}(1-\alpha) \), \( A_2 \) becomes negative and this couple will never be concerned with social sterility. This is due to a simple income effect: the higher the income of the husband, the more the couple is protected against situations of childlessness or constrained fertility due to poverty. In the extreme case where \( w_m^I \geq \frac{2\alpha}{(2\alpha-1)\hat{\theta}} \), only Regime VI exists and regimes of fertility with Malthusian properties disappear.

The present proof does not necessarily hold when the assumption on \( \theta(w_f^I, w_m^I) \) fixed is relaxed. Indeed, equations involving \( \theta \) (like Equations (12) to (14)) would not behave in the same way (for instance, their left or right hand sides would not be strictly concave or convex). This could induce the existence of more thresholds on \( a \) and \( w_f^I \). Nevertheless, intuitively, we can say that for \( \theta \to 1 \), our results hold as \( \theta \) would vary only weakly around one half.

We propose in the next subsection to verify if the results of our proposition hold for the values of parameters provided in Table 3. We can see that despite the estimated value of \( \hat{\theta} \) is not close to one, our proposition still holds. In a weak sense, this allows us to affirm that there exists a non empty set of parameters of the model for which our proposition holds even if \( \theta \) is not constant.

Interpretations:

Figures 20, 21 and 22 and following paragraphs describe the main results of the proposition. For very low non-labor income \( (a < A_0) \) and, low \( w_m^I \), a couple will be childless whatever the wife’s wage. The wage allowing the couple to have children is, in such a case, higher than the wage for which they would choose to be childless (Regime IX).
Figure 20: Fertility Conditional on Being Married when $a \in [A_0, A_1]$ and when $a \in [A_1, A_2]$

Figure 21: Fertility Conditional on Being Married when $a \in [A_2, A_3]$ and when $a \in [A_3, A_4]$

Figure 22: Fertility Conditional on Being Married when $a \in [A_4, A_5]$ and when $a > A_5$
For higher levels of non-labor income \((a > A_0)\), having children becomes feasible and optimal for some wages. Figure 20 (left) shows that, when \(a \in [A_0, A_1]\), a couple can have children in the constrained fertility regime (Regime VIII) but the unconstrained regime (Regime VI) is never optimal. In Regime VIII, the husband reduces his consumption in order to enable his wife to have children. As shown in Figure 20 (right), Regime VI becomes optimal for some wages and \(a > A_1\). In Figure 20, the substitution effect is dominated in the constrained fertility regime and an increase in \(w^f\) increases fertility.

\(W^f_A\) corresponds to the wage allowing couples to procreate. For a wage greater than but close to \(W^f_A\), the consumption of the husband is close to zero, implying that parenthood is not optimal. This explains why Regime IX always precedes Regime VIII in Figure 20 and Figure 21 (left).

It can be shown that \(W^f_D\) decreases with \(a\). This implies that higher levels of non-labor income move the fertility hump in Figure 20 leftwards. This means that a couple with higher non-labor income can reach the unconstrained fertility regime (Regime VI) for smaller \(w^f\).32

In Figures 21 and 22, \(a > \hat{c} - (1 - \phi(1 - \alpha)\eta)w^m\). Social sterility disappears as a couple can always ensure a consumption level of \(\hat{c}\) to the wife. It also implies that fertility decreases with \(w^f\) in Regime VIII because the substitution effect dominates. When \(a \in [A_3, A_4]\) and \(w^f\) is low, a couple has to choose between two extreme cases: Regime X where the wife specializes in child rearing and both spouses have a low consumption level; and Regime IX where she specializes in labor market activities and is childless. In other words, the couple has to choose between having a maximum of children but lacking of consumption goods and consuming more but lacking of children.33 When \(w^f\) is very low, the surplus from consumption that could come from her labor market activity is also very low and the couple chooses to have the maximal number of children. When her wage becomes greater than \(W^f_C\), the surplus from consumption provides a higher marginal utility than the surplus from children. Then, the couple becomes childless in Regime IX. Finally, for higher wages, Regime VIII dominates the previous two.

Regimes X and XI, where the wife has the highest fertility appear in Figures 21 (right) and 22, for low wages. In Regime X, the wife consumes \(\hat{c}\) and specializes in childrearing. This regime becomes feasible once \(a > A_2\), but is optimal for some wages only when \(a \geq A_3\) (when \(a < A_3\) the consumption of the husband is too low). Once \(w^f\) becomes sufficiently

32 Using as a proxy for wealth the fact of owning or renting the dwelling, we observe that those who rent the housing unit have larger childlessness rates, for all levels of education. This is supported by Figure 20. We can also observe that the decreasing relationship between completed fertility of mothers and education moves leftwards. This is also in line with Figures 20, 21 and 22 as \(W^f_D\) decreases with \(a\).

33 When \(w^f \in [0, W^f_C]\), Regime VIII is always dominated by at least one of these two extreme regimes.
high, the wife still consumes \( \hat{c} \) but spends a part of her time on labor market activities. The additional income earned by the couple is allocated to the consumption of the husband. Finally, Figure 22 (right) exhibits the same fertility pattern as Figure 19 for a single woman and the same intuitions apply.

Poor couples will then be either childless or in a regime where the wife has the highest fertility. In all cases, couples in which the wife is highly educated choose to remain childless.

**B.4 Fertility as a function of \( w^f, a \) and \( w^m \) (married)**

Figure 23 allows to see how the various 2d plots of the main text for married women are slices for a given level of \( a \) of a 3d picture. The parameters are set equal to those estimated in Subsection 4.2. Three different levels of \( w^m \) are shown.

![Figure 23: Fertility for different levels of mother’s education \((w^f)\) and non-labor income \((a)\). Father with no school (left), Grade 12 (middle), Doctoral degree (right)](image)

**B.5 Marriage Decisions**

We now turn to the first stage of the game and consider the decision to accept a marriage offer from a randomly drawn person of the opposite sex. Once this potential partner has been drawn, we know the vector \((w^f, a^f, w^m, a^m)\). To understand how potential spouses decide to accept or reject a marriage offer, we first analyze their behavior in a fictive situation where they are sure their partner is not naturally sterile. Then, we describe how the risk of natural sterility modifies these behaviors.

**Behaviors without natural sterility**

Notice that here, we are able to provide some elements to highlight this choice but we cannot perform a complete inspection as comparing indirect utilities in all the cases would not be analytically tractable.
Let us first consider the potential bride. From Propositions 1 and 2 we know the two relevant regimes she has to compare. When the potential bride has both a low wage and a low non-labor income, she could either be in Regime II or in Regime III if she remains single. If the relevant regime for the woman when single is Regime II, then she always prefers to be married with a husband allowing her to consume at least \( \hat{c} \), as:

\[
u(c_{II}, 0) < \min\{u(\hat{c}, n_{VIII}), u(c_{VII}, 0), u(\hat{c}, \bar{n}_M), u(c_{XI}, \bar{n}_M)\}.
\] (15)

She will reject a marriage offer only if she is in Regime VII if married and if \( u(c_{II}, 0) > u(c_{VII}, 0) \), that is:

\[
(1 - \delta_f)w_f + a_f - \mu > \theta(w_f, w^m)(w^m + w_f + a).
\] (16)

The only motive for a very poor woman to marry a man who does not provide enough income to allow her to procreate lies in the scale economies, in goods and in time, allowed by marriage. Moreover, an uneducated woman will accept to marry an uneducated man because similar education ensures that negotiation powers remain close to one half which allows the surplus from marriage to be shared equally.

If the relevant regime for a woman when single is Regime III, she always prefers to be a single mother who consumes \( \hat{c} \) rather than a childless married woman who consumes less than \( \hat{c} \) (i.e. \( u(\hat{c}, n_{III}) > u(c_{VII}, 0) \)). Hence, she would always reject marriage offers leading to Regime VII. Moreover, \( u(\hat{c}, n_{III}) < \min\{u(\hat{c}, \bar{n}_M), u(c_{XI}, \bar{n}_M)\} \), since in Regimes X and XI the woman would consume at least \( \hat{c} \) and would have the maximum number of children. So she would always accept marriage offers leading to Regimes X and XI.

For higher \( w_f \) and \( a_f \), Regimes I, IV or V are accessible as a single person. When a woman earns high wages, Regime IV does not dominate in only two circumstances: (a) when the additional income coming from the man is not high enough to incite the woman to become a mother but sufficiently close to her own income for the marriage surplus to be shared (Regime IX) and (b) when the man’s wage is high enough to incite her to become a mother (Regime VI or XI).

We can conclude that, unlike highly educated women, lowly educated women will accept almost any match on the marriage market. This highlights the role of marriage as an institution protecting women against poverty, social sterility and even against living in a Malthusian like regime. Highly educated women do not need to be protected against social sterility by marriage. On the contrary, for highly educated women, being matched with highly educated men is the occasion to have children rather than being single and childless because marriage reduces the opportunity cost of motherhood.
As being single is not only a personal choice, let us now turn to the problem of the potential husband. A single man has the following indirect utility,

\[ \bar{u}^m = \ln \left( (1 - \delta^m)w^m + a^m - \mu \right) + \ln \nu. \]

Getting married is the only way for him to become a father but entails two costs: an opportunity cost due to the time spent with children\(^{34}\) and a potential decrease of his consumption. This implies that a man will agree to marry without having children only if it increases his own consumption, which arises when the potential bride has enough education. Regimes VII and IX can then dominate singleness for the man.

The opportunity to become a father is not always sufficient to incite the man to marry: if having children decreases his consumption too sharply, he will remain single (singleness can dominate Regimes VI, VIII, X and XI). This could apply to a man with a low income, matched with a woman who would be socially sterile as single: in order to have children in Regime VIII, he would then have to give up too much of his income and consume too little.

Once again, marriage is a protection against social sterility: two partners could have children by pooling their income and their time while they would have remained childless and single otherwise. Men who live in Regime VIII agree to reduce their consumption in order to enable their wives to have children. This is optimal if \( \hat{c} \) is not too large and the man not too poor. Marriage can even prevent poor women becoming poor single mothers. This is the case when a highly educated man agrees to marry a poorly educated woman: even if her negotiation power is minimal (\( \theta = \frac{1}{2} \theta \)), she will consume more than \( \hat{c} \) because consumption increases with \( w^m \).

The parameter \( \alpha \) is important for marital decisions. For high values of \( \alpha \), the model predicts higher marriage rates for highly educated men than for highly educated women as better educated men have more reasons to marry: they can have children at a low opportunity cost. For highly educated women the incentive to marry is now low and marrying a lowly educated man would only entail costs (they would remain childless whatever the marital status). Poor women, on the contrary, will marry more often as they are less often rejected by highly educated men who can have children without reducing their consumption too much. For low values of \( \alpha \), the opportunity cost of becoming a father is high and nothing ensures that highly educated men marry more often than highly educated women. The parameters \( \delta^i \)'s also affect the marital choice. A large \( \delta^m \) implies that educated men are more willing to accept a marriage offer, in line with the male marriage pattern stressed in Section 2.

\(^{34}\)In the case where \( (1 - \alpha)\phi n > \delta^m - \phi \eta (1 - \alpha) \), his time dedicated to labor market activities is smaller when married than when single.
The model predicts a positive degree of assortativeness due to the parameter $\theta$. A higher $\theta$ implies that spouses’ incomes are more equally shared. An individual with a much larger income than his or her potential spouse will then be more likely to reject the offer as he/she would have to give up too much of his/her own consumption.

To conclude, the model predicts lower female marriage rates for extreme social classes. Women with low levels of education are more likely to be socially sterile while highly educated women will probably want to remain childless. This makes women from extreme social classes less attractive to men. Marriage could however occur when wages and non-labor incomes are sufficiently close for them to share the marriage surplus.

**Natural sterility**

The possibility of being naturally sterile or being matched with a naturally sterile partner has an impact on the incentive to marry. As couples face a higher risk of being sterile than single women, natural sterility tends *ceteris paribus* to reduce the overall marriage rate. In particular, the number of marriages between highly educated men and poorly educated women are affected. Indeed, an incentive for highly educated men to marry a lowly educated woman lies in the possibility of becoming a father. Now, when they marry a poor woman, they have to reduce their consumption and to take the risk of remaining childless. Women with high education levels are less concerned by this phenomenon as they can have children on their own.

A high probability of sterility among men will discourage almost all types of women to accept a marriage offer. Indeed, for a men, being rich remains a sufficient condition to convince a very poor woman who would have been subject to social sterility to accept a marriage offer as it offers to this woman a possibility to have children and to increase her consumption (condition (15) remains valid). Nevertheless, for other women, the higher the probability of male sterility, the weaker the incentive to accept a marriage offer. Obviously, as the probability of natural sterility is the same among all men, rich ones still have an advantage on the marriage market.
C Simulation and Robustness

C.1 Implementation Notes

Minimization was first run using the algorithm PIKAIA and the results used as initial values in UOBYQA. PIKAIA is a genetic algorithm developed by Charbonneau (2002), it allows global extrema to be found in highly non-linear optimization problems where there exist a high number of local extrema. We used PIKAIA in a first step to identify the region in the parameter space where the global maximum lies. Once this region has been identified, we used a faster algorithm called UOBYQA (Powell (2002)) designed to find a maximum of a well-behaved problem. It was developed for optimization when first derivatives of the objective function were not available and takes account of the curvature of the objective function, by forming quadratic models by interpolation. We ran these two algorithms in FORTRAN 90.

C.2 Alternative Measures of Sterility

In a previous version of the paper (see Baudin et al. (2012)), we followed an alternative where natural sterility equals 3.7%. This number is obtained from Leridon (2008) who uses the Henry database, which consists of data for rural France from the 18th century, when fertility control was ineffective. Restricting the sample to couples where the husband and the wife were still living together at age 50, 3.7% of women who married at age 20-24 remained childless. The main problem from this study is that living conditions and bad nutrition at that time might positively influence childlessness.

women with 9 years of schooling have the lowest rate of childlessness, 1.26% (2,626 married women were childless among 208,761 married women). However, this number might underestimate natural sterility. A first reason for this is that it may be affected by divorce rates caused by fecundity problems. Another possible argument is that sterile Vietnamese couples adopt orphans and then answer to the number of children they have ever born as if these children were biologically theirs.

C.3 Non-labor income

In the main text, we assume that non-labor income does not depend on the education of individuals. Here, we relax this hypothesis with the following expression for $a^i$:

$$a^i = \beta w^i + \tilde{a}^i$$  \hspace{1cm} (17)

where $\beta$ describes the marginal impact of education on non-labor income (bequests and ones education might have common determinants, such as the education of the parents, see Hertz et al. (2007)). $\tilde{a}^i$ is the part of non labor income that is uncorrelated to education and differs for each individual: each adult draws a different $\tilde{a}^i$ from a log-normal distribution, whose mean and variance will be estimated as before. We then estimate the same model adding one parameter to the estimation, $\beta$. Results are provided in Table 18. We see that the estimates are similar comparing both specifications and that the estimated value for $\beta$ is zero. Hence, the three facts exposed in Section 2 do not allow to identify an effect of education on income other than the one coming through the labor income.

C.4 Computation of the Standard Errors

Two methods are used in the literature to obtain the standard errors of parameters estimated by the simulated method of moments: bootstrapping and the delta method. The static nature of the model makes the bootstrapping method preferable to the delta method which is usually faster but tends to underestimate the standard errors. We first draw 200 random new samples with replacement from the original data. Each new bootstrap sample is of equal size of the original one (1,127,080 observations) but the frequency of each observation changes. For each of these new datasets we generate the 72 moments and estimate the corresponding parameters. We then compute the standard errors of these estimators. By doing so, the uncertainty surrounding the estimated parameters comes exclusively from the uncertainty around the estimated moments. The results are reported in Column (2) of Table 19, and in Table 3 of the main text.
We expected the uncertainty coming from the randomness of the artificial population used to simulate the model to be minimal when drawing a large enough number of individuals (typically, it is 100,000 per education category). To check this expectation we also provide in Column (1) of Table 19 the parameters when the only uncertainty comes from the model: that is when we estimate the parameters 200 times using the same empirical moments but drawing different households from the distribution. Column (3) of Table 19 presents the parameters when we combine both uncertainties, using the empirical moments from the bootstrap samples and drawing different households from the distribution. The difference between Columns (2) and (3) is very small.

C.5 Identified Parameters for Subsamples

Identification by Race or Ethnicity. In the main text we have assumed an homogeneous marriage market. Here we assume instead that there are fragmented markets for each race separately (see Appendix A.5.1 for how the groups are constructed). We therefore reestimate the parameters for each race independently. The results are provided in Table 20.

Some parameters show remarkable constancy across races, such as the mean of the non labor income $m_a$, the rearing cost $\phi$ and $\eta$, the bargaining power parameter $\theta$. There are strong variations in the $\delta^i$'s across races. Black single men loose 34.9% of their time endowment compared to married black men, while this loss equals 25.6% in the whole population. Asian single men only loose 19.2%. Black fathers are characterized by a lower involvement in childrearing. Non-labor income is less dispersed for Whites and more dispersed for Blacks and Hispanics.

The breakdown of childlessness into its components shows that blacks are relatively more affected by social sterility, while Asians are more childless due to a higher opportunity cost in than all the other groups.

Identification Removing Disabled. Since 83.5% of single childless women with no schooling are disabled, we also identify the parameters of the model after removing the disabled from the data. The results are shown in Table 21. We can check that the parameter $\hat{c}$ still plays a role and that its value is not significantly different from its benchmark estimate. The estimated social sterility drops with respect to the benchmark.

C.6 Alternative Specification for $\theta$

In this appendix, we provide results under two alternative specifications for the bargaining power $\theta$. The first fixes the bargaining power to $1/2$, independently of relative education
levels. The second makes bargaining depend on relative potential incomes, hence including non labor incomes $a^f$ and $a^m$.

First, assume $\vartheta = 1$ (equal negotiation power of spouses). Table 22 reports the implied parameter values. The objective $f(p)$ and the $R^2$ deteriorate. The cost of marriage drastically increases for a highly educated individual matched with someone with a low education: he/she will have to give up a large part of his/her income to ensure equal consumption in the household. As a result, highly educated will reject unions with lowly educated more often and the percentage of married people in the extreme education categories decreases. As heterogamous unions become less valuable, marriage rates also decrease on average. But this is partly compensated by a rise in the parameter $\delta^m$. Finally, as poor women are less often married, they are also less protected against social sterility when $\vartheta = 1$. Then the estimates of both types of childlessness increase.

Second, we include non-labor incomes into the bargaining power of each spouse as follows:

$$\theta(w^f, w^m, a^f, a^m) = \frac{1}{2} \theta + (1 - \theta) \frac{w^f + a^f}{w^f + w^m + a^f + a^m}$$

This specification leads to the results shown in Table 22. We see that most of the parameters are significantly different but that the match of the model with the data is still very good. Some moments are better matched with one specification of $\theta$ while others with the other.

### C.7 Changes in the Time Allocation Parameter $\alpha$

The model assumes that the share that mothers ($\alpha$) and fathers ($1-\alpha$) devote to childrearing is constant across education groups and over time. Bianchi et al. (2004) show that the ratio of married mothers’ to married fathers’ time in child care declined between the mid-1960s and the late 1990s, which could be an indication that either social norms changed for all education categories, or that the increase in the education of women makes it optimal for couples to have the father more involved in the raising of children.

To have some insights on the role of $\alpha$ we carried out the following two experiments. First, fixing $\alpha = 1$ instead of identifying it from the cross sectional data, and re-estimating the rest of the parameters, we find that the quality of the match is lower: the model becomes unable to reproduce (a) a reasonable marriage rate (especially for highly educated women who have lost their incentive to marry), (b) childlessness rates for highly educated married women (for whom the cost of raising children becomes extremely high so they have more incentives to be childless in [O]), (c) the gap between the fertility of married mothers and
that of single mothers, who now face the same opportunity cost. Hence, allowing $\alpha < 1$ is important to generate the nice features of the model.

Second, consider that couples set $\alpha$ optimally under the constraint $\alpha \in [1/2, 1]$. A full-fledged model with proper bargaining on $\alpha$ would be the topic of another paper; however, a simple benchmark would be to assume the following rule:

$$
\alpha = \begin{cases} 
1 & \text{if } w_f < w_m \\
1/2 & \text{if } w_f \geq w_m.
\end{cases}
$$

With this specification the marriage rates are reasonable, the U-shaped relationship between childlessness and education of married women is preserved, but the model fails in reproducing the high fertility of poorly educated married mothers (by about one child for the two lowest education categories), as poor married mothers face almost the same incentives as poor single mothers. Otherwise, the simple ad-hoc rule above does rather well, which indicates that bargaining over $\alpha$ could be a promising extension. Finally, we also made $\alpha$ depend on the education of the mother, independently of the education of the father: $\alpha = 1 - w_f/2$.

The properties of this simulation are similar to those in the previous case.

### C.8 Assortative Matching

The model accounts for 13.8% of the variation in the assortativeness of matching. In reality, the assortativeness is higher because, first, people meet several possible matches (we would need a dynamic model to reproduce this), and second, individuals are more likely to meet others who have similar levels of education to their own.

With a model able to match the level of assortative matching, in particular among poor people, we expect a lower $\hat{c}$. In fact, 53% of women with no education marry a man with no education. This is much higher than in our model. Consequently, our estimated value of $\hat{c}$ is probably too high because we have more women with no education marrying men with higher levels of education than in reality. This means that in order to match observed childlessness, $\hat{c}$ must be high. A way to increase the degree of endogenous assortativeness consists in modifying the bargaining rule (4) by making it less sensitive to the wage ratio. This will increase the rejection rate of matches with very different people. However, achieving more assortativeness will be at the expense of matching a reasonable marriage rate, as people only meet once in our static set-up. This is a limitation of our approach.

A complementary and exogenous way to generate the right degree of assortativeness is to assume, as in Fernández-Villaverde et al. (2010), that a fraction $\lambda$ of the female population
draws a possible match from her education category, while $1 - \lambda$ draws from the total population. For each chosen value of $\lambda$, we can estimate the remaining parameters and see whether changes are important.\textsuperscript{35} We provide the results in Table 25. Our main results are robust to this change. For instance, when $\lambda$ equals 0.1, the fit of the model does not change significantly although the marriage matrix becomes much more satisfying.

The number of marriages by education category from the data are given in Table 23. This table is constructed in the following way: first, we drop from the data all the individuals who are not married (MARST > 1) or who do not have an identified partner in the Census (SPLOC= 0). Then, we sort observations, first, by their serial number, corresponding to the household, and then by their sex, so that the man of the household comes before his wife in the data. We then generate a variable saying that the husband has a corresponding wife after him (the serial number for both has to be the same). The last step is to generate a variable with the education of the husband and another variable with the education of the wife. The ratio
\[
\frac{\pi(i, j)}{\sum_j \pi(i, j)}
\]
gives the proportion of men of type $j$ having married a woman of type $i$, where $\pi(i, j)$ is the number of couples in which the woman has an education level $i$ and the man an education level $j$. Dividing this number by the proportion of women of type $i$ in the total population,
\[
\frac{\sum_i \pi(i, j)}{\sum_{i,j} \pi(i, j)},
\]
we obtain the data in Table 24. Each cell gives, for each married man, his increased chances marrying a woman in a given category of education, compared with a purely random matching framework. If there was no assortativeness, all the cells would equal 1. The first cell means that a man of category 1 has 57 times more chances of marrying a woman of education category 1 than in the case of pure random matching. We are able to compute similar statistics with the simulated data. These are provided at the bottom of Table 24. We see that the model generates some assortative matching endogenously, although lower than in the data. Regressing each cell in the top part of the table on the corresponding cell in the bottom part leads to an $R^2$ of 0.138 meaning that we are able to account for 13.8% of the variations in the assortativeness matching.

To assess the influence of having abstracted from exogenous assortative matching on the estimated parameters, we conducted the following exercise. A share, $\lambda$, of the population

\textsuperscript{35}In Fernández-Villaverde et al. (2010), a fraction of the population is randomly matched, the rest being matched following a Gale-Shapley algorithm, which generates perfect assortativeness.
draws a potential spouse from his/her education category. The remaining share $1 - \lambda$ draws his/her partner randomly from the whole population. For different values of $\lambda$, we reestimate the parameters, minimizing the same objective as before. Table 25 presents the results. The case $\lambda = 0$ refers to the benchmark case of Table 3. The parameters which seems to be the most sensitive to changing $\lambda$ are $\alpha$ and $\theta$: these are significantly different from the benchmark for all the different given values of $\lambda$.

Table 25 also provides the fit of the model with respect to each of the moments used for estimating the parameters. These $R^2$ are computed as follows:

$$R^2 = 1 - \frac{\sum_{k=1}^{12} (d_k - s_k)^2}{\sum_{k=1}^{12} (d_k - \bar{d}_k)^2}$$

where $d_k$ and $s_k$ are respectively the empirical and simulated moments for each education category $k$. Comparing the different $R^2$, we see that the simulated moments are quite insensitive to the choice of $\lambda$. For childlessness rates and fertility of single women, the best specification is the benchmark, for the fertility of married women, it is $\lambda = 0.2$ or $\lambda = 0.3$, and for marriage rates, $\lambda = 0.4$.

The value of $\lambda$ that minimizes the objective function $f(p)$ is 0.1. For $\lambda = 0.1$, we obtain the marriage matrix presented in Table 26. Regressing now each cell in the top part of Table 24 on the corresponding cell of Table 26 leads to an $R^2$ of 0.784. We conclude that introducing some exogenous assortative matching, captured by $\lambda > 0$, allows us to get much closer to the observed assortativeness, without modifying the estimated parameters or the ability of the model to reproduce the targeted empirical moments very much.
<table>
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<th>Parameter</th>
<th>Value with $\beta = 0$</th>
<th>Value with $\beta &gt; 0$</th>
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</thead>
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Table 18: Identified Parameters with $\beta > 0$

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<th>Both (3)</th>
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<td>s.e.</td>
<td>mean</td>
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Table 19: Mean and Standard Errors of Parameters
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<th>Natives</th>
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<th>Hispanics</th>
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</table>

| [O] | 8.1% | 6.1% | 7.3% | 3.4% | 10.4% | 5.8% |
| [S] | 2.4% | 5.1% | 5.8% | 1.8% | 2.3% | 3.4% |
| [N] | 2.3% | 2.3% | 2.3% | 2.4% | 2.3% | 2.3% |

Table 20: Identified Parameters by Race or Ethnicity

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<tbody>
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<tr>
<td>$\delta^f$</td>
<td>0.077</td>
<td>0.085</td>
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| [O] | 8.1% | 8.0% |
| [S] | 2.4% | 2.1% |
| [N] | 2.3% | 2.3% |

* indicates a significant difference from “All”.

Table 21: Identified Parameters Without the “Disabled”
### Table 22: Identified Parameters for Alternative Specifications of the Bargaining Power

<table>
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<th>Parameter</th>
<th>$\theta(w^f, w^m)$</th>
<th>$\theta = 1/2$</th>
<th>$\theta(w^f, w^m, a^f, a^m)$</th>
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<td>0.426</td>
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<td>0.912</td>
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<td>0.950</td>
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<td>1.002</td>
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<tr>
<td>$\hat{c}$</td>
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<tr>
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$^*$ indicates a significant difference from the benchmark $\theta(w^f, w^m)$. 

Table 22: Identified Parameters for Alternative Specifications of the Bargaining Power
Table 23: Marriages per Education Category. Men in Columns, Women in Rows

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Table 23: Marriages per Education Category. Men in Columns, Women in Rows
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Table 24: Assortative Matching in the Data and Generated by the Model with $\lambda = 0$, Men's Education in Columns, Women's in Rows
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<td>marriage female</td>
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<td>0.455</td>
<td>0.555</td>
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<tr>
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<td>0.655</td>
<td>0.605</td>
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<td>$\sigma_a$</td>
<td>0.247</td>
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<td>0.252</td>
<td>0.284*</td>
<td>0.281*</td>
<td>0.284*</td>
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<tr>
<td>$\bar{m}_a$</td>
<td>1.001</td>
<td>1.002</td>
<td>0.999</td>
<td>0.990*</td>
<td>1.003</td>
<td>1.037</td>
</tr>
<tr>
<td>$\hat{c}$</td>
<td>0.399</td>
<td>0.386</td>
<td>0.360*</td>
<td>0.336*</td>
<td>0.332*</td>
<td>0.308*</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.272</td>
<td>0.253</td>
<td>0.264</td>
<td>0.252</td>
<td>0.257</td>
<td>0.284</td>
</tr>
<tr>
<td>$\bar{\theta}$</td>
<td>0.864</td>
<td>0.908*</td>
<td>0.898*</td>
<td>0.920*</td>
<td>0.990*</td>
<td>0.950*</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.524</td>
<td>0.504*</td>
<td>0.500*</td>
<td>0.503*</td>
<td>0.500*</td>
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</tr>
<tr>
<td>$\phi$</td>
<td>0.206</td>
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<td>0.218*</td>
<td>0.221*</td>
<td>0.224*</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.114</td>
<td>0.120*</td>
<td>0.109</td>
<td>0.120*</td>
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<td>0.109</td>
</tr>
<tr>
<td>$\delta^m$</td>
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<td>0.287*</td>
<td>0.262</td>
<td>0.254</td>
<td>0.278</td>
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</tr>
<tr>
<td>$\delta^f$</td>
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<td>0.090</td>
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<td>0.076</td>
<td>0.048*</td>
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</table>

* indicates a significant difference from the case $\lambda = 0$.

Table 25: Identified Parameters for $\lambda$ between 0 and 0.5
Table 26: Simulated Marriage Matrix for $\lambda = 0.1$, Men's Education in Columns, Women's in Rows

<table>
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<th></th>
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<th>2</th>
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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
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<td>0.914</td>
<td>0.907</td>
<td>0.892</td>
<td>0.894</td>
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<td>0.841</td>
<td>0.73</td>
</tr>
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<td>0.919</td>
<td>0.909</td>
<td>0.903</td>
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<td>0.894</td>
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<td>0.908</td>
<td>0.902</td>
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<td>0.909</td>
<td>0.901</td>
<td>0.894</td>
<td>0.898</td>
<td>0.875</td>
<td>0.879</td>
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<tr>
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<td>0.938</td>
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<td>0.896</td>
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<td>0.893</td>
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<tr>
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<td>1.061</td>
<td>1.123</td>
<td>8.754</td>
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</tbody>
</table>

Additional References


